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轴向可伸缩悬臂复合材料层合板横向振动的解析研究*

高艳红[1†] 张伟[1] 吕书锋[2]

(1.北京工业大学 材料与制造学部,北京 100124)

(2.内蒙古工业大学 理学院,呼和浩特 010051)

摘要 通过解析和数值方法对可伸缩悬臂复合材料层合板的时变动力学特性进行研究.根据经典层合板理论和 Hamilton 原理对面内激励和横向载荷共同作用下的可伸缩悬臂复合材料层合板进行线性动力学建模,选取符合可伸缩悬臂板位移边界条件的时变模态函数,利用 Galerkin 方法对所得的偏微分方程进行离散,得到复合材料层合板的时变常微分线性动力学方程.研究轴向移动速度对可伸缩悬臂板动力学特性的影响,并通过改进的多尺度法得到了一阶时变线性系统的解析解和数值解比较.结果表明,轴向移动速度对可伸缩悬臂板的动力学特性影响很大;相比文献[12],本文采取的改进的多尺度法对一阶线性时变系统更高效.

关键词 可伸缩悬臂板, 线性时变系统, 拓展的多尺度法, 解析研究

中图分类号:TB33;O321 **文献标志码**:A

An Analytical Study on Transverse Vibration of Axially Telescopic Cantilever Composite Laminates*

Gao Yanhong[1†] Zhang Wei[1] Lv Shufeng[2]

(1. Faculty of Materials and Manufacturing, Beijing University of Technology, Beijing 100124, China)

(2. College of Science, Inner Mongolia University of Technology, Hohhot 010051, China)

Abstract The time-varying dynamic characteristics of telescopic cantilever composite laminates were studied by analytical and numerical methods. According to the classical laminate plate theory and Hamilton principle, a model of the telescopic cantilever composite laminate under in-plane excitation and lateral load was established. By choosing suitable time-varying modal functions, partial differential equations were discretized by Galerkin method. The time-varying ordinary differential linear dynamics equation is obtained. The effect of axially moving speed on dynamic characteristics of the telescopic cantilever plate is studied. The analytical solution obtained by an improved multi-scale method and numerical solution of the first-order time-varying linear system are compared. The results show that the axial speed has great influence on the dynamic characteristics of the telescopic cantilever plate. Compared with reference [12], the improved multi-scale method adopted in this paper is more efficient for first-order linear time-varying systems.

Key words telescopic cantilever plate, linear time-varying dynamics, extended multiple-scale method, analytical study

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† 通信作者 E-mail:gydxgyh@163.com

引言

近年来，轴向可伸缩悬臂结构引起了广大研究者的广泛关注.这类结构主要应用于可伸缩机械手臂、大型可展天线、可伸缩机翼和导弹等.其中，可伸缩机翼具有外伸时展弦比增大，巡航和着陆性能好，而当机翼回收时，展弦比降低，阻力小，俯冲速度快等优点.但是，这类结构在轴向运动中极易发生失稳和破坏；而且，可伸缩悬臂结构在轴向运动的过程中质量、刚度、阻尼及边界条件都是变化的，对其研究难度较大，因此，对可伸缩悬臂结构的动力学研究具有非常重要的理论意义和工程应用价值.

目前，很多学者采用数值方法对可伸缩悬臂结构进行研究[1-3].但是，很少有人对可伸缩悬臂结构采用解析方法进行研究，其中，最常用的解析方法有多尺度法、平均法等.Öz 等[4]和 Parker 等[5]开展了带有时变速度的轴向运动简支梁的非线性振动和稳定性研究，并通过多尺度法得到时变系统近似解析解.Chen 等[6]运用 Lindstedt-Poincare 方法研究非线性时变系统解析解.李山虎等[7]用多尺度法和数值法研究低速轴向外伸悬臂梁的动力学特性.Yang 等[8]研究多频激励下的轴向运动梁的近似解析解.Liu 等[9]通过平均法对轴向移动梁的动力学问题进行了解析研究.Yang 等[10]采用假设参数法得到不同边界下的轴向移动梁的固有频率和近似解，结果表明当弯曲刚度梁以较大速度外伸时，通过此种方法所得解析解有很高精确性.Mahdi 等[1]提出控制假设模态法研究带有移动接头的轴向运动柔性梁的动力学问题.Yang 等[12]应用改进的多尺度法、谐波平衡和平均法对具有变质量和变刚度的线性时变系统进行解析研究.Wang 等[13]对湿热环境下具有不同边界条件的轴向移动悬臂梁的非线性自由振动特性进行研究，并用多尺度法得到了非线性无阻尼自由振动的频率解析表达式.除此之外，崔云涛[14]对可伸缩翼进行了实验研究，结果表明可伸缩翼的轴向移动速度与最大响应频率呈线性关系.段应昌等[15]通过修正的 Galerkin 方法和实验研究了可伸缩悬臂梁的横向振动特性.Yang 等[16]通过数值方法进行可伸缩悬臂梁的能量分析，结果表明考虑一阶初始条件下可伸缩悬臂梁的主要能量集中在第一阶模态，距离初始条件阶数较近的模态获得的能量较多，距离越远能量越少.

本文将可伸缩翼模拟成可伸缩悬臂复合材料层合板，并研究其在匀速外伸和回收过程的时变动力学特性.根据经典层合板理论和 Hamilton 原理对受面内激励和横向激励联合作用的可伸缩悬臂复合材料层合板进行建模，然后，通过选取满足时变位移边界条件的模态函数进行一阶 Galerkin 离散得到一阶时变常微分方程.最后，通过改进的多尺度法和数值方法分析轴向移动速度对可伸缩悬臂板的动力学特性的影响.

1 基本方程

考虑受面内激励和一阶气动力联合作用的可伸缩悬臂复合材料层合矩形板，面内激励的表达式为 $f_y = f_0 + f_1\cos(\omega t)$，一阶气动力表达式为 Δp，该板沿着 x 轴运动的速度为 $v = v_0 + v_d\cos(\omega t)$，其中 $f_1\cos(\omega t)$，$v_d\cos(\omega t)$ 是扰动项.板的初始长度为 l_0，板的总长度为 l，板宽为 b，板的厚度为 h，如图 1 所示.

根据经典层合板理论[17]，可伸缩悬臂复合材料层合板的位移场可以写为

$$u(x,y,t) = u_0(x(t),y,t) - z\frac{\partial w_0}{\partial x} \tag{1a}$$

$$v(x,y,t) = v_0(x(t),y,t) - z\frac{\partial w_0}{\partial y} \tag{1b}$$

$$w(x,y,t) = w_0(x(t),y,t) \tag{1c}$$

其中，u_0, v_0, w_0 分别表示笛卡儿直角坐标系上沿着 x, y, z 方向的位移，$z\dfrac{\partial w_0}{\partial x}$，$z\dfrac{\partial w_0}{\partial y}$ 分别表示沿着 x, y 方向的分量.

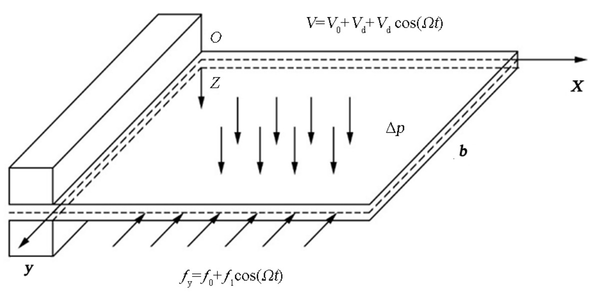


图 1 可伸缩悬臂板模型

Fig.1 The model of telescopic cantilever

位移-应变关系可以写为

$$\varepsilon_{xx} = \frac{\partial u}{\partial x}, \varepsilon_{yy} = \frac{\partial v}{\partial y}, \varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \tag{2}$$

根据 Hamilton 原理可得线性动力学方程：

$$N_{xx,x}+N_{xy,y}=I_0\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+I_0\ddot{u}_0-I_1\frac{\partial^2}{\partial t^2}\left(\frac{\partial w_0}{\partial x}\right) \tag{3a}$$

$$N_{yy,y}+N_{xy,x}=I_0\ddot{v}_0-I_1\frac{\partial^2}{\partial t^2}\left(\frac{\partial w_0}{\partial y}\right) \tag{3b}$$

$$\frac{\partial^2 M_{xx}}{\partial x^2}+\frac{\partial^2 M_{yy}}{\partial y^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}+\Delta p-\gamma\dot{w}_0= I_0\ddot{w}_0+I_1\left(\frac{\partial\ddot{u}_0}{\partial x}+\frac{\partial\ddot{v}_0}{\partial y}\right)-I_2\left(\frac{\partial^2\ddot{w}_0}{\partial x^2}+\frac{\partial^2\ddot{w}_0}{\partial y^2}\right) \tag{3c}$$

其中

$$I_i=\int_{-h/2}^{h/2}z^i\rho(z)\mathrm{d}z\quad(i=0,1,2) \tag{4}$$

根据一阶活塞理论[18]，一阶气动力的表达式为

$$\Delta p=-\frac{4q_\mathrm{d}\lambda}{M_\infty}\left(\frac{1}{\upsilon}\frac{\partial w_0}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t}+\frac{\partial w_0}{\partial y}+\frac{1}{\upsilon}\frac{\partial w_0}{\partial t}\right) \tag{5}$$

其中 $q_\mathrm{d}=1/2\rho_\mathrm{a}\upsilon^2$ 代表动压，ρ_a 表示空气密度，υ 是来流速度，M_∞ 和 λ 表示马赫数和动力修正因子：$\lambda=M_\infty/\sqrt{M_\infty^2-1}$.可伸缩悬臂板在固定和自由端的边界条件为

$$\text{当 } x=0,\ u_0=v_0=w_0=0 \tag{6a}$$

$$\text{当 } x=l(t),\ N_{xx}=N_{xy}=M_{xx}=M_{xy}=0 \tag{6b}$$

$$\text{当 } y=0,b,\ N_{xy}=M_{yy}=M_{xy}=0 \tag{6c}$$

正交铺设复合材料层合板的应变位移-关系为

$$\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\varepsilon_{xx}^{(0)}\\ \varepsilon_{yy}^{(0)}\\ \gamma_{xy}^{(0)}\end{Bmatrix}+z\begin{Bmatrix}\varepsilon_{xx}^{(1)}\\ \varepsilon_{yy}^{(1)}\\ \gamma_{xy}^{(1)}\end{Bmatrix} \tag{7}$$

其中

$$\begin{Bmatrix}\varepsilon_{xx}^{(0)}\\ \varepsilon_{yy}^{(0)}\\ \gamma_{xy}^{(0)}\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial u_0}{\partial x}\\ \dfrac{\partial v_0}{\partial y}\\ \dfrac{\partial u_0}{\partial y}+\dfrac{\partial v_0}{\partial x}\end{Bmatrix},\quad \begin{Bmatrix}\varepsilon_{xx}^{(1)}\\ \varepsilon_{yy}^{(1)}\\ \gamma_{xy}^{(1)}\end{Bmatrix}=\begin{Bmatrix}-\dfrac{\partial^2 w_0}{\partial x^2}\\ -\dfrac{\partial^2 w_0}{\partial y^2}\\ -2\dfrac{\partial^2 w_0}{\partial x\partial y}\end{Bmatrix} \tag{8}$$

正交各向异性层合板的内力和力矩与应变的关系为

$$\begin{Bmatrix}N_x\\ N_y\\ N_{xy}\end{Bmatrix}=\begin{bmatrix}A_{11}&A_{12}&0\\ A_{21}&A_{22}&0\\ 0&0&A_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x^{(0)}\\ \varepsilon_y^{(0)}\\ \gamma_{xy}^{(0)}\end{Bmatrix} \tag{9a}$$

$$\begin{Bmatrix}M_x\\ M_y\\ M_{xy}\end{Bmatrix}=\begin{bmatrix}D_{11}&D_{12}&0\\ D_{21}&D_{22}&0\\ 0&0&D_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x^{(1)}\\ \varepsilon_y^{(1)}\\ \gamma_{xy}^{(1)}\end{Bmatrix} \tag{9b}$$

层合板的刚度系数为

$$(A_{ij},D_{ij})=\int_{-h/2}^{h/2}Q_{ij}(1,z^2)\mathrm{d}z\quad(i,j=1,2,6) \tag{10}$$

将方程(9)代入方程(3)，可以得到以广义位移关系表示的可伸缩悬臂复合材料层合板的线性动力学方程为

$$A_{11}\frac{\partial u_0^2}{\partial x^2}+(A_{12}+A_{66})\frac{\partial^2 v_0}{\partial x\partial y}+A_{66}\frac{\partial^2 u_0}{\partial y^2}= I_0\ddot{x}(t)+I_0\frac{\partial^2 u_0}{\partial t^2}-I_1\frac{\partial^2}{\partial t^2}\left(\frac{\partial w_0}{\partial x}\right) \tag{11a}$$

$$A_{22}\frac{\partial^2 v_0}{\partial y^2}+(A_{12}+A_{66})\frac{\partial^2 u_0}{\partial x\partial y}+A_{66}\frac{\partial^2 v_0}{\partial x^2}= I_0\frac{\partial^2 v_0}{\partial t^2}-I_1\frac{\partial^2}{\partial t^2}\left(\frac{\partial w_0}{\partial y}\right) \tag{11b}$$

$$\begin{aligned}&D_{11}\frac{\partial^4 w_0}{\partial x^4}+D_{22}\frac{\partial^4 w_0}{\partial y^4}+(D_{12}+D_{21}+4D_{66})\frac{\partial^4 w_0}{\partial x^2\partial y^2}+\frac{4q_d\lambda}{M_\infty}\frac{\partial^2 w_0}{\partial y^2}+f_y\frac{\partial^2 w_0}{\partial y^2}=-I_0\ddot{w}_0-\\&\left(\gamma+\frac{4q_d\lambda}{M_\infty}\frac{1}{v_a}\right)\dot{w}_0-\frac{4q_d\lambda}{M_\infty}\frac{1}{v_a}\frac{\partial w_0}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t}+\\&I_2\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 w_0}{\partial x^2}+\frac{\partial^2 w_0}{\partial y^2}\right)-I_1\frac{\partial^2}{\partial t^2}\left(\frac{\partial u_0}{\partial x}+\frac{\partial v_0}{\partial y}\right)\end{aligned} \tag{11c}$$

考虑到可伸缩悬臂板面内振动较小，因此忽略 u 和 v 方向的位移，只考虑 w 方向的位移，为了满足可伸缩悬臂板的位移边界条件，选取悬臂梁和自由梁沿 x 和 y 方向的模态函数进行研究，如下所示：

$$w(x,y,t)=\sum_{i=1}^{n}w_i(t)X_i(x)Y_j(y)\quad(i=1,2,\cdots,n) \tag{12a}$$

$$X_i(x,t)=\sin\frac{k_ix}{l(t)}-\sinh\frac{k_ix}{l(t)}+\alpha_i\left(\cosh\frac{k_ix}{l(t)}-\cos\frac{k_ix}{l(t)}\right) \tag{12b}$$

$$Y_i(y)=\sin\frac{u_iy}{b}+\sinh\frac{u_iy}{b}-\beta_i\left(\cosh\frac{u_iy}{b}+\cos\frac{u_iy}{b}\right) \tag{12c}$$

为了对方程(11c)进行无量纲化，引入的无量纲参数，如下所示：

$$\bar{w}_0=\frac{w_0}{h},\ \bar{x}=\frac{x}{l_0},\ \bar{y}=\frac{y}{b},\ \bar{\Omega}=\frac{1}{\pi^2}\left(\frac{l_0b\rho}{E}\right)^{1/2}\Omega$$

$$\bar{t}=\pi^2\left(\frac{E}{l_0b\rho}\right)^{1/2}t,\ \bar{A}_{ij}=\frac{(l_0b)^{1/2}}{Eh^2}A_{ij}$$

$$\bar{D}_{ij}=\frac{(l_0 b)^{1/2}}{Eh^4}D_{ij}\ ,\ \bar{I}_i=\frac{1}{(l_0 b)^{(i+1)/2}\rho}I_i$$

$$\bar{\gamma}=\frac{1}{l_0 b}\sqrt{\frac{1}{\rho E}}\ \gamma\ ,\ \bar{f}=\frac{b^2}{Eh^3}f \tag{13}$$

忽略面内振动和惯性项的影响,可得无量纲化的方程.

$$c_{10}\frac{\partial^4 w_0}{\partial x^4}+c_{11}\frac{\partial^4 w_0}{\partial y^4}+c_{12}\frac{\partial^4 w_0}{\partial x^2\partial y^2}+ c_{13}\left(\frac{\partial w_0}{\partial x}\right)^2\frac{\partial^2 w_0}{\partial x^2}+c_{14}\left(\frac{\partial w_0}{\partial y}\right)^2\frac{\partial^2 w_0}{\partial y^2}+ c_{15}\left(\frac{\partial w_0}{\partial x}\right)^2\frac{\partial^2 w_0}{\partial y^2}+c_{16}\left(\frac{\partial w_0}{\partial y}\right)^2\frac{\partial^2 w_0}{\partial x^2}+ c_{17}\frac{\partial^2 w_0}{\partial x\partial y}\frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y}+c_{18}\frac{\partial^2 w_0}{\partial y^2}+c_{19}\frac{\partial w_0}{\partial x}+ c_{20}\frac{\partial w_0}{\partial y}+c_{21}\frac{\partial w_0}{\partial t}=c_{22}\frac{\partial^2 w_0}{\partial t^2} \tag{14}$$

其中,为了简化,在推导过程中略去了变量符号中的横线.将时变模态函数(12)代入方程(14),进行伽辽金离散,可得一阶线性常微分方程,如下所示.

$$\beta_1\ddot{w}_1+\beta_2\dot{w}_1+\beta_3 w_1=0 \tag{15}$$

其中 β_i 是关于时间的函数.

2 拓展的多尺度法

根据文献[11],选取的时间尺度为

$$t\rightarrow\{t_1,t_2\},t_1=t,t_2=\frac{1}{\varepsilon}\int_0^t\omega(t)\mathrm{d}t \tag{16}$$

其中 $\omega(t)=\sqrt{\beta_3/\beta_1}$.

对时间的两阶微分算子为

$$\frac{\mathrm{d}}{\mathrm{d}t}=\frac{\partial}{\partial t_1}+\varepsilon\frac{\partial}{\partial t_2}\omega(t) \tag{17a}$$

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}=\frac{\partial^2}{\partial t_1^2}+\frac{1}{\varepsilon}\left[\dot{\omega}\frac{\partial}{\partial t_2}+2\omega\frac{\partial^2}{\partial t_1\partial t_2}\right]+\frac{1}{\varepsilon^2}\omega^2\frac{\partial^2}{\partial t_2^2} \tag{17b}$$

将方程(17)代入方程(15),忽略阻尼项,根据的指数项相同可得

$$\varepsilon^0:\quad\frac{\partial^2 w_1}{\partial t_2^2}+w_1=0 \tag{18a}$$

$$\varepsilon^1:\quad\dot{\omega}\frac{\partial w_1}{\partial t_2}+2\omega\frac{\partial^2 w_1}{\partial t_1\partial t_2}=0 \tag{18b}$$

设方程(18a)的解为

$$w(t_1,t_2)=a(t_1)\cos(t_2+\beta_0) \tag{19}$$

将方程(19)代入方程(18b)可得

$$a(t_1)=a(0)\sqrt{\frac{\omega(0)}{\omega(t)}} \tag{20}$$

当 $0<\varepsilon\ll 1$ 时,将方程(18)代入方程(17)可得一阶线性时变方程的近似解析解

$$w=a(0)\sqrt{\frac{\omega(0)}{\omega(t)}}\cos\left[\frac{1}{\varepsilon}\int_0^t\omega(t)\mathrm{d}t+\beta_0\right] \tag{21}$$

其中, $\omega(0)$ 是结构的初始固有频率, $\beta_0=\arctan(\omega(0)w(0)/\dot{w}(0))$ 表示初始相位, $a(0)=\sqrt{w(0)^2+(w(0)/\omega(0))^2}$ 是结构的初始振幅.与文献[11]不同的是,考虑当 $\varepsilon=1$ 时,式(15)的近似解析解为

$$w=a(0)\sqrt{\frac{\omega(0)}{\omega(t)}}\cos\left[\int_0^t\omega(t)\mathrm{d}t+\varphi_0\right] \tag{22}$$

其中

$$\varphi_0=-\left[\int_0^t\omega(t)\mathrm{d}t\right]\Big|_{t=0}+\beta_0.$$

3 对比验证

为了研究可伸缩悬臂复合材料层板的动力学特性,板的尺寸和材料参数如下所示: $l_0=1.5\mathrm{m}$, $b=1.5\mathrm{m}$, $l=4\mathrm{m}$, $f_0=2000\ \mathrm{N/m^2}$, $\Omega=10$, $\rho_a=0.65\ \mathrm{kg/m^3}$, $M_a=3.0$, $\upsilon=900\mathrm{m/s}$, $E_1=125.0\mathrm{GPa}$, $E_2=7.2\mathrm{GPa}$, $G_{23}=1.43\mathrm{GPa}$, $G_{12}=G_{13}=4.1\mathrm{GPa}$, $\upsilon_{12}=0.33$, $\upsilon_{21}=\upsilon_{12}E_2/E_1$, $\rho=1570\mathrm{kg/m^3}$, $\gamma=300\mathrm{Ns/m}$.根据方程(15)和方程(22),比较了一阶线性数值解和近似解析解,并分析轴向移动速度对板的动力学特性的影响.

板以不同的轴向移动速度匀速外伸和回收过程中的时间历程,如图2所示.其中,蓝色线代表数值解,红色代表解析解.从图2可以看出,板在轴向移动过程中的数值解和解析解非常吻合,因此可以证明拓展的多尺度法求解一阶线性时变系统解析解是可行的.除此之外,由图2(a)可以看出,板在外伸过程中,频率减小,振幅增大,这与回收过程图2(d)是相反的;并且,由图2(a)~图2(c)可以看出,随着轴向外伸速度逐渐增大,板的频率逐渐减小,但是振幅变化不大,这与回收过程图2(d)~2(f)的现象相同.结果表明,轴向移动速度对可伸缩悬臂板的振动频率影响很大.

当可伸缩悬臂板轴向匀速外伸和匀速回收时不同轴向移动速度对板的固有频率的影响,如图3所示.其中图3(a)和(b)分别是外伸和回收过程轴向移动速度对板的固有频率的影响,外伸速度分别为0.1m/s,0.5m/s,1m/s,2m/s和5m/s;回收速度

分别为 0.1m/s,0.2m/s,0.3m/s,0.4m/s 和 0.5m/s.从图 3 可以看出轴向移动速度越大,可伸缩悬臂板的频率变化越快,这与图 2 的结论是一致的.这表明轴向移动速度对板的频率影响较大.

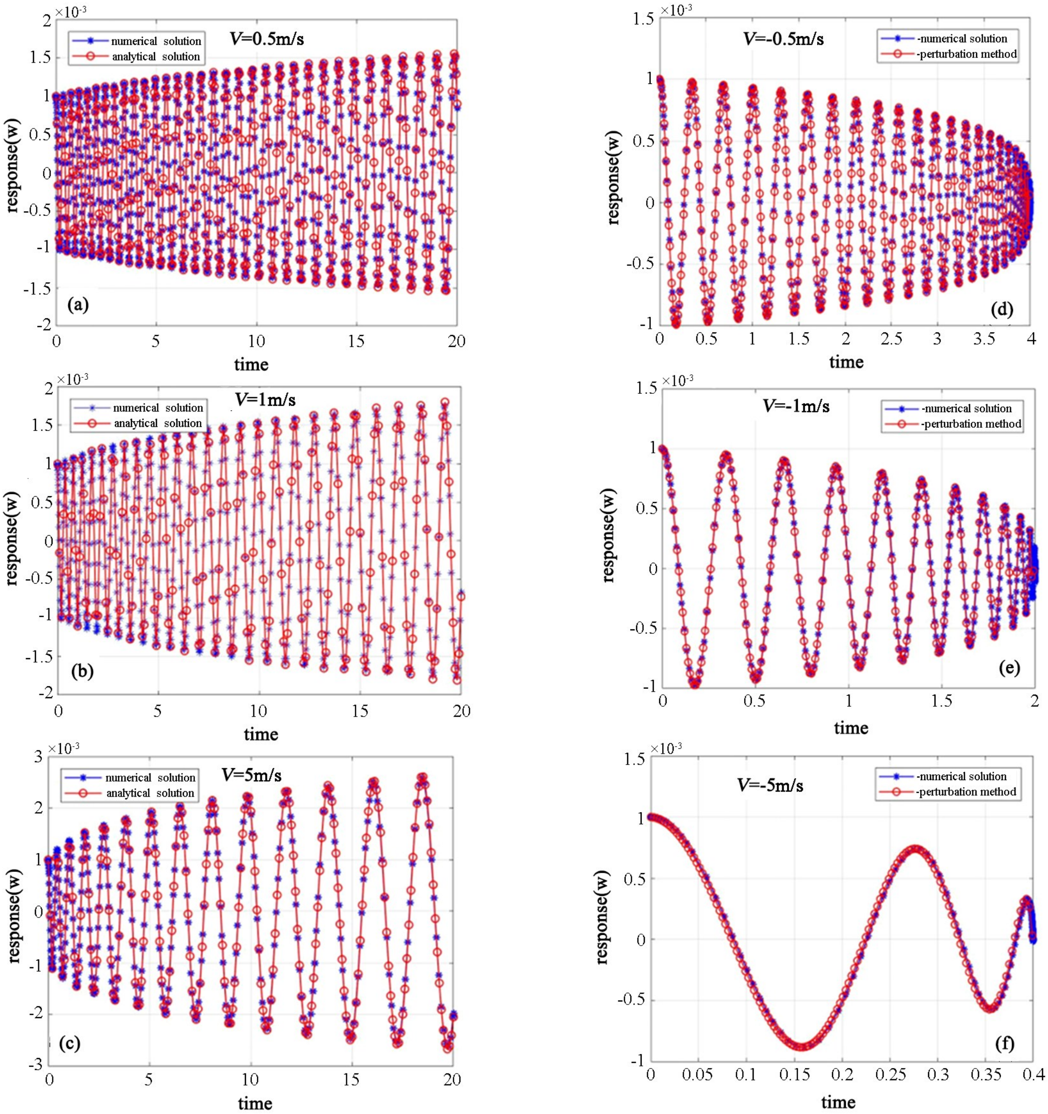
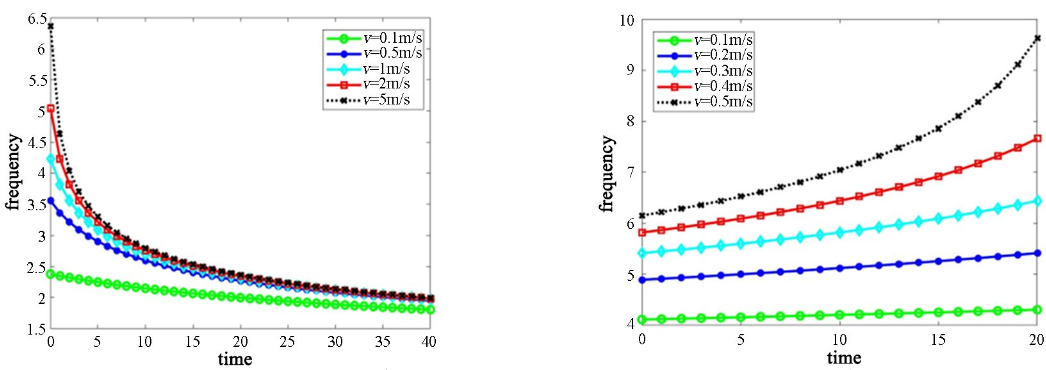


图 2　当可伸缩悬臂板以不同轴向速度匀速外伸和回收时的时间历程图:(a),(b),(c) 当外伸速度 0.5m/s,1m/s 和 5m/s 时板的时间历程图,(d),(e),(f) 当回收速度为 0.5m/s,1m/s 和 5m/s 时板的时间历程图

Fig.2　Time history curves of the telescopic cantilever plate when the plate is deploying and retracting at different uniform speeds: (a)(b)(c), the time history curves of the plate when the deploying speed is 0.5m/s,1m/s and 5m/s, (d)(e)(f), the time history curves of the plate when the retracting speed is 0.5m/s,1m/s and 5m/s



(a)外伸过程中固有频率的变化　　(b) 回收过程中固有频率的变化

(a)The natural frequency varies in deploying process　　(b)The natural frequency varies in retracting process

,图 3　当可伸缩悬臂板匀速外伸和回收时轴向移动速度对板固有频率的影响

Fig.3　The effect of axially moving speed on natural frequency of the telescopic cantilever plate when the plate is deploying and retracting at different uniform speeds

4 结论

本文将可伸缩机翼模拟成可伸缩悬臂复合材料层合矩形板模型.通过经典剪切理论和 Hamilton 原理建立了线性动力学方程.通过解析和数值研究方法研究了可伸缩悬臂板在一阶气动力和面内激励联合作用下的动力学特性,分析轴向移动速度对板在匀速外伸和回收过程中振幅和振动频率的影响,同时通过拓展的多尺度法得到了一阶线性时变系统的近似解析解,结果表明拓展的多尺度法对时变系统动力学研究是可行的.除此之外,轴向移动速度对可伸缩悬臂板的动力学特性影响很大,无论板是处于外伸还是回收过程,轴向移动速度增大都会使振动频率的变化加快.

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