串联非线性能量阱的高分支响应研究*

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摘要 本文对比研究了单自由度 NES 和两自由度串联 NES 的吸振效能,重点分析了串联 NES 对高分支响应的抑制作用.运用复变量-平均法,推导出连接串联 NES 的线性主振子的慢变方程,获得了系统的频率响应曲线.在不同激励幅值下,将连接单自由度 NES 与连接串联两自由度 NES 的系统响应进行对比,结果表明,串联 NES 能够在较大,幅值激励范围内保持较高吸振效能.在两种幅值激励下,分析了串联 NES 的参数对减振效果的影响.研究结果表明,在小幅激励时,调整一级 NES 参数对主振子的响应影响较为明显;而在大幅激励时,二级 NES 参数对减振效果影响较大.

关键词 非线性能量阱, 大幅激励, 高分支响应, 吸振效能, 串联

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引言

非线性能量阱(nonlinear energy sink,NES)是一种具有立方刚度的强非线性吸振器,具有质量轻及宽频带吸振等优点[1-3],近年来受到了学者们的广泛关注.然而,单自由度非线性能量阱的吸振效能仍然相对较低,更严重的是,在大幅简谐激励下,该吸振器较易引发主振系统的分叉,进而产生系统的大幅振动.因此,学者们提出了多种改进方案,其中,串联非线性能量阱是较易实现的一种方案,且已证明,在冲击载荷下,其吸振效能明显高于单自由度非线性能量阱.Tsakirtzis 等[4]研究了两自由度串联 NES 在冲击激励下的吸振效能,研究表明,串联 NES 比单自由度 NES 拥有更强的靶向能量传递能力与更广的能量吸收频带,在一定条件下,其能量吸收率可达 90% 以上.Gendelman 等[5,6]也在研究串联 NES 中发现,通过对 NES 引入内部自由度,即额外串联一级 NES,使其之间能产生一种协同效应,从而可以增大 NES 靶向能量传递的范围.而这一协同效应在并联 NES 中同样存在,其中机理也得到理论分析与实验验证[7,8].

陈恒等[9]利用单自由度 NES 实现对机翼的颤振控制.Starosvetsky 和 Gendelman 对连接 NES 的耦合系统产生的强调制响应(SMR)进行了研究,并指出在强调制响应阶段 NES 的吸振效率较高[10-12].而张也弛等[13]也在简谐激励下验证了强调制响应利于 NES 吸振,且在不增加额外质量的前提下,连接串联 NES 的主振子相比于连接单自由度 NES 的主振子拥有更高的减振效果.另外,在连接串联 NES 的系统中,强调制响应也可通过调整一、二级 NES 质量分布来产生,从而达到减振的目的[14].而针对两自由度 NES 中串联方式与并联方式的选择,李继伟等[15]提出了一套选择方法,即在主振子初始位移较大时可选择串联 NES,并选择较弱刚度,反之则选择并联 NES.

单自由度 NES 对于初始能量具有选择性,吸振效果也受初始能量变化影响较大甚至会因输入能量过大而失效[16],在简谐激励下,NES 的失效则具体表现在激励幅值增大后,耦合系统出现了高分支响应,对此,Starosvetsky 等[17]通过设置分段二次非线性阻尼消除高分支响应,Chen 等[18]尝试通过并联 NES 提高高分支响应发生的激励幅值.

目前,对于串联 NES 的研究仍不充分,仍未见有关于串联 NES 所引发的系统高分支响应方面的

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研究.本文重点研究串联NES在简谐激励幅值大范围变化下的吸振效能.首先运用复变量-平均法得出系统的慢变力学方程,求解并判断解的稳定性,并使用龙格-库塔法证明该近似系统的正确性.其次,在不同简谐激励幅值下研究了连接串联NES的耦合系统的响应,并与单自由度NES的振动抑制效果进行对比.最后,在大、小两种激励下,研究串联NES中两级NES的参数对其吸振效果的影响并对其进行优化.

1 系统模型

单自由度主振子和串联两自由度NES所构成的系统如图1所示,该耦合系统在简谐激励下的动力学方程由牛顿第二定律可得:

$$\begin{aligned}&\ddot{w}+\gamma_0\dot{w}+k_0w+k_{nl1}(w-v_1)^3+\\&\quad\gamma_1(\dot{w}-\dot{v}_1)=A\cos\Omega t\\&\varepsilon_1\ddot{v}_1-k_{nl1}(w-v_1)^3-\gamma_1(\dot{w}-\dot{v}_1)+\\&\quad k_{nl2}(v_1-v_2)^3+\gamma_2(\dot{v}_1-\dot{v}_2)=0\\&\varepsilon_1\varepsilon_2\ddot{v}_2-k_{nl2}(v_1-v_2)^3-\gamma_2(\dot{v}_1-\dot{v}_2)=0\end{aligned}\qquad(1)$$

w,v_1,v_2分别表示主振子、一级NES和二级NES的位移,γ_0,γ_1与γ_2分别表示主振子、一级NES和二级NES的阻尼系数,k_0为主振子刚度,k_{nl1}与k_{nl2}表示一级NES与二级NES的非线性刚度.ε_1表示一级NES与主振子的质量比,ε_2表示一级NES与二级NES的质量比,且满足$\varepsilon_1\ll1$;A表示简谐激励幅值,Ω表示激励频率.为分析方便,令$m_0=1$.

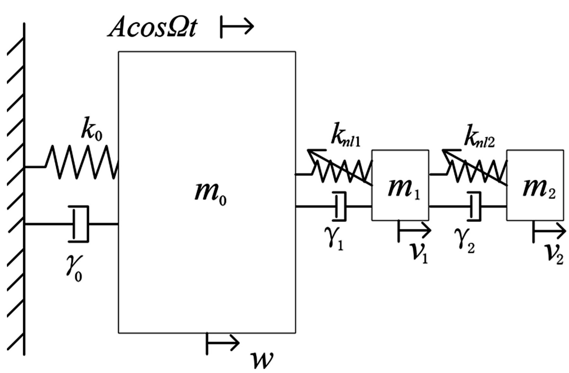

图1 耦合系统结构示意图

Fig.1 Structural representation of the coupled system

引入如下变量代换,

$$u_1=w-v_1,\ u_2=v_1-v_2,$$

$$\dot{w}+i\Omega w=\alpha_1e^{i\Omega t},\ \dot{w}-i\Omega w=\bar{\alpha}_1e^{-i\Omega t},$$

$$\dot{u}_1+i\Omega u_1=\alpha_2e^{i\Omega t},\ \dot{u}_1-i\Omega u_1=\bar{\alpha}_2e^{-i\Omega t},$$

$$\dot{u}_2+i\Omega u_2=\alpha_3e^{i\Omega t},\ \dot{u}_2-i\Omega u_2=\bar{\alpha}_3e^{-i\Omega t}\qquad(2)$$

其中,$\alpha_1,\alpha_2,\alpha_3$为时间$t$的函数,$\bar{\alpha}_1,\bar{\alpha}_2,\bar{\alpha}_3$表示$\alpha_1,\alpha_2,\alpha_3$的共轭,$i$为虚数单位,由式(2)可得:

$$w=\frac{\alpha_1e^{i\Omega t}-\bar{\alpha}_1e^{-i\Omega t}}{2i\Omega},\ \dot{w}=\frac{\alpha_1e^{i\Omega t}+\bar{\alpha}_1e^{-i\Omega t}}{2},$$

$$\ddot{w}=\frac{1}{2}(\dot{\alpha}_1e^{i\Omega t}+i\Omega\alpha_1e^{i\Omega t}+\dot{\bar{\alpha}}_1e^{-i\Omega t}-i\Omega\bar{\alpha}_1e^{-i\Omega t}),$$

$$u_1=\frac{\alpha_2e^{i\Omega t}-\bar{\alpha}_2e^{-i\Omega t}}{2i\Omega},\ \dot{u}_1=\frac{\alpha_2e^{i\Omega t}+\bar{\alpha}_2e^{-i\Omega t}}{2},$$

$$\ddot{u}_1=\frac{1}{2}(\dot{\alpha}_2e^{i\Omega t}+i\Omega\alpha_2e^{i\Omega t}+\dot{\bar{\alpha}}_2e^{-i\Omega t}-i\Omega\bar{\alpha}_2e^{-i\Omega t}),$$

$$u_2=\frac{\alpha_3e^{i\Omega t}-\bar{\alpha}_3e^{-i\Omega t}}{2i\Omega},\dot{u}_2=\frac{\alpha_3e^{i\Omega t}+\bar{\alpha}_3e^{-i\Omega t}}{2},$$

$$\ddot{u}_2=\frac{1}{2}(\dot{\alpha}_3e^{i\Omega t}+i\Omega\alpha_3e^{i\Omega t}+\dot{\bar{\alpha}}_3e^{-i\Omega t}-i\Omega\bar{\alpha}_3e^{-i\Omega t})\qquad(3)$$

将式(3)带入方程(1),并消除快变项,可得慢变方程如下:

$$\begin{aligned}&\dot{\alpha}_1+i\Omega\alpha_1+\gamma_0\alpha_1+\frac{k_0\alpha_1}{i\Omega}+\frac{3k\alpha_2^2\bar{\alpha}_2}{4i\Omega^3}+\gamma_1\alpha_2=A\\&\varepsilon_1(\dot{\alpha}_1+i\Omega\alpha_1-\dot{\alpha}_2-i\Omega\alpha_2)-\frac{3k_{nl1}\alpha_2^2\bar{\alpha}_2}{4i\Omega^3}-\\&\quad\gamma_1\alpha_2+\frac{3k_{nl2}\alpha_3^2\bar{\alpha}_3}{4i\Omega^3}+\gamma_2\alpha_3=0\\&\varepsilon_2(\dot{\alpha}_1+i\Omega\alpha_1-\dot{\alpha}_2-i\Omega\alpha_2-\dot{\alpha}_3-i\Omega\alpha_3)-\\&\quad\frac{3k_{nl2}\alpha_3^2\bar{\alpha}_3}{4i\Omega^3}-\gamma_2\alpha_3=0\end{aligned}\qquad(4)$$

令$\alpha_1=a_1+ib_1,\alpha_2=a_2+ib_2,\alpha_3=a_3+ib_3$,其中$a_1,b_1,a_2,b_2,a_3,b_3$均为时间$t$的函数.将其代入方程(4),分离实部和虚部,可得:

$$\dot{a}_1=b_1\Omega-a_1\gamma_0-\frac{k_0b_1}{\Omega}-\frac{3k_{nl1}b_2(a_2^2+b_2^2)}{4\Omega^3}-\gamma_1a_2+A$$

$$\dot{b}_1=-a_1\Omega-b_1\gamma_0+\frac{k_0a_1}{\Omega}+\frac{3k_{nl1}a_2(a_2^2+b_2^2)}{4\Omega^3}-\gamma_1b_2$$

$$\dot{a}_2=A-a_1\gamma_0+b_2\Omega-\frac{k_0b_1}{\Omega}-a_2\gamma_1-\frac{3k_{nl1}b_2(a_2^2+b_2^2)}{4\Omega^3}-\left(\frac{1}{\varepsilon_1}\right)\cdot\left(\gamma_1a_2+\frac{3k_{nl1}b_2(a_2^2+b_2^2)}{4\Omega^3}-\frac{3k_{nl2}b_3(a_3^2+b_3^2)}{4\Omega^3}-\gamma_2a_3\right)$$

$$\dot{b}_2=-b_1\gamma_0-a_2\Omega+\frac{k_0a_1}{\Omega}-b_2\gamma_1+\frac{3k_{nl1}a_2(a_2^2+b_2^2)}{4\Omega^3}-\left(\frac{1}{\varepsilon_1}\right)\cdot\left(\gamma_1b_2-\frac{3k_{nl1}a_2(a_2^2+b_2^2)}{4\Omega^3}+\frac{3k_{nl2}a_3(a_3^2+b_3^2)}{4\Omega^3}-\gamma_2b_3\right)$$

$$\dot{a}_3=-a_3\Omega+\left(\frac{1}{\varepsilon_1\varepsilon_2}\right)\left(\frac{3k_{nl2}a_3(a_3^2+b_3^2)}{4\Omega^3}-b_3\gamma_2\right)+\left(\frac{1}{\varepsilon_1}\right)\cdot\left(\gamma_1b_2-\frac{3k_{nl1}a_2(a_2^2+b_2^2)}{4\Omega^3}+\frac{3k_{nl2}a_3(a_3^2+b_3^2)}{4\Omega^3}-\gamma_2b_3\right)$$

$$\dot{b}_3=b_3\Omega-\left(\frac{1}{\varepsilon_1\varepsilon_2}\right)\left(\frac{3k_{nl2}b_3(a_3^2+b_3^2)}{4\Omega^3}+a_3\gamma_2\right)+\left(\frac{1}{\varepsilon_1}\right)\cdot\left(\gamma_1a_2+\frac{3k_{nl1}b_2(a_2^2+b_2^2)}{4\Omega^3}-\frac{3k_{nl2}b_3(a_3^2+b_3^2)}{4\Omega^3}-\gamma_2a_3\right)$$

(5)

为获得系统的稳态响应,令 $\dot{a}_1=0,\dot{b}_1=0,\dot{a}_2=0,\dot{b}_2=0,\dot{a}_3=0,\dot{b}_3=0$,可得到描述稳态响应的非线性代数方程组:

$$b_1\Omega-a_1\gamma_0-\frac{k_0b_1}{\Omega}-\frac{3k_{nl1}b_2(a_2^2+b_2^2)}{4\Omega^3}-\gamma_1a_2+A=0$$

$$-a_1\Omega-b_1\gamma_0+\frac{k_0a_1}{\Omega}+\frac{3k_{nl1}a_2(a_2^2+b_2^2)}{4\Omega^3}-\gamma_1b_2=0$$

$$A-a_1\gamma_0+b_2\Omega-\frac{k_0b_1}{\Omega}-a_2\gamma_1-\frac{3k_{nl1}b_2(a_2^2+b_2^2)}{4\Omega^3}-\left(\frac{1}{\varepsilon_1}\right)\cdot\left(\gamma_1a_2+\frac{3k_{nl1}b_2(a_2^2+b_2^2)}{4\Omega^3}-\frac{3k_{nl2}b_3(a_3^2+b_3^2)}{4\Omega^3}-\gamma_2a_3\right)=0$$

$$-b_1\gamma_0-a_2\Omega+\frac{k_0a_1}{\Omega}-b_2\gamma_1+\frac{3k_{nl1}a_2(a_2^2+b_2^2)}{4\Omega^3}-\left(\frac{1}{\varepsilon_1}\right)\cdot\left(\gamma_1b_2-\frac{3k_{nl1}a_2(a_2^2+b_2^2)}{4\Omega^3}+\frac{3k_{nl2}a_3(a_3^2+b_3^2)}{4\Omega^3}-\gamma_2b_3\right)=0$$

$$-a_3\Omega+\left(\frac{1}{\varepsilon_1\varepsilon_2}\right)\left(\frac{3k_{nl2}a_3(a_3^2+b_3^2)}{4\Omega^3}-b_3\gamma_2\right)+\left(\frac{1}{\varepsilon_1}\right)\cdot\left(\gamma_1b_2-\frac{3k_{nl1}a_2(a_2^2+b_2^2)}{4\Omega^3}+\frac{3k_{nl2}a_3(a_3^2+b_3^2)}{4\Omega^3}-\gamma_2b_3\right)=0$$

$$b_3\Omega-\left(\frac{1}{\varepsilon_1\varepsilon_2}\right)\left(\frac{3k_{nl2}b_3(a_3^2+b_3^2)}{4\Omega^3}+a_3\gamma_2\right)+\left(\frac{1}{\varepsilon_1}\right)\cdot\left(\gamma_1a_2+\frac{3k_{nl1}b_2(a_2^2+b_2^2)}{4\Omega^3}-\frac{3k_{nl2}b_3(a_3^2+b_3^2)}{4\Omega^3}-\gamma_2a_3\right)=0$$

(6)

求解非线性代数方程组(6),可以获得主振子的振幅:

$$A_w=\frac{\sqrt{a_1^2+b_1^2}}{\Omega} \tag{7}$$

为了判断系统稳态响应的稳定性,令:

$$a_1=a_{10}+\delta_1,\ b_1=b_{10}+\delta_2$$
$$a_2=a_{20}+\delta_3,\ b_2=b_{20}+\delta_4$$
$$a_3=a_{30}+\delta_5,\ b_3=b_{30}+\delta_6 \tag{8}$$

其中,$a_{10},b_{10},a_{20},b_{20},a_{30},b_{30}$ 是方程(6)的稳态解,$\delta_n(n=1,2,3,4,5,6)$ 表示稳态解的小扰动,将式(8)代入方程组(5),并且仅保留关于扰动的线性部分,可得:

$$\dot{\delta}_1=-\gamma_0\delta_1+\Omega\delta_2-\frac{k_0}{\Omega}\delta_2-\frac{3k_{nl1}}{4\Omega^3}\cdot[(a_{20}^2+3b_{20}^2)\delta_4+2a_{20}b_{20}\delta_3]-\gamma_1\delta_3$$

$$\dot{\delta}_2=-\gamma_0\delta_2-\Omega\delta_1+\frac{k_0}{\Omega}\delta_1+\frac{3k_{nl1}}{4\Omega^3}\cdot[(3a_{20}^2+b_{20}^2)\delta_3+2a_{20}b_{20}\delta_4]-\gamma_1\delta_4$$

$$\dot{\delta}_3=-\gamma_0\delta_1-\frac{k_0}{\Omega}\delta_2-\left(1+\frac{1}{\varepsilon_1}\right)\left(\frac{3k_{nl1}a_{20}b_{20}}{2\Omega^3}+\gamma_1\right)\delta_3+\left[\Omega-\left(1+\frac{1}{\varepsilon_1}\right)\frac{3k_{nl1}(a_{20}^2+3b_{20}^2)}{4\Omega^3}\right]\delta_4+\frac{1}{\varepsilon_1}\cdot\left(\frac{3k_{nl2}a_{30}b_{30}}{2\Omega^3}+\gamma_2\right)\delta_5+\frac{1}{\varepsilon_1}\frac{3k_{nl2}(a_{30}^2+3b_{30}^2)}{4\Omega^3}\delta_6$$

$$\dot{\delta}_4=\frac{k_0}{\Omega}\delta_1-\gamma_0\delta_2+\left(1+\frac{1}{\varepsilon_1}\right)\left(\frac{3k_{nl1}(3a_{20}^2+b_{20}^2)}{4\Omega^3}-\Omega\right)\delta_3+\left(1+\frac{1}{\varepsilon_1}\right)\left(\frac{3k_{nl1}a_{20}b_{20}}{2\Omega^3}-\gamma_1\right)\delta_4-\frac{1}{\varepsilon_1}\frac{3k_{nl2}(3a_{30}^2+b_{30}^2)}{4\Omega^3}\delta_5-\frac{1}{\varepsilon_1}\left(\frac{3k_{nl2}a_{30}b_{30}}{2\Omega^3}-\gamma_2\right)\delta_6$$

$$\dot{\delta}_5=\frac{1}{\varepsilon_1}\left(\frac{3k_{nl1}a_{20}b_{20}}{2\Omega^3}+\gamma_1\right)\delta_3+\frac{1}{\varepsilon_1}\frac{3k_{nl1}(a_{20}^2+3b_{20}^2)}{4\Omega^3}\delta_4-\frac{1}{\varepsilon_1}\left(1+\frac{1}{\varepsilon_2}\right)\left(\frac{3k_{nl2}a_{30}b_{30}}{2\Omega^3}+\gamma_2\right)\delta_5+\left[\Omega-\frac{1}{\varepsilon_1}\left(1+\frac{1}{\varepsilon_2}\right)\frac{3k_{nl2}(a_{30}^2+3b_{30}^2)}{4\Omega^3}\right]\delta_6$$

$$\dot{\delta}_6=-\frac{1}{\varepsilon_1}\frac{3k_{nl1}(3a_{20}^2+b_{20}^2)}{4\Omega^3}\delta_3+\frac{1}{\varepsilon_1}\left(\gamma_1-\frac{3k_{nl1}a_{20}b_{20}}{2\Omega^3}\right)\delta_4+\left[\frac{1}{\varepsilon_1}\left(1+\frac{1}{\varepsilon_2}\right)\frac{3k_{nl2}(3a_{30}^2+b_{30}^2)}{4\Omega^3}-\Omega\right]\delta_5+\frac{1}{\varepsilon_1}\left(1+\frac{1}{\varepsilon_2}\right)\left(\frac{3k_{nl2}a_{30}b_{30}}{2\Omega^3}-\gamma_2\right)\delta_6 \tag{9}$$

利用方程(9)的系数矩阵,求得其特征根以判断稳态解的稳定性.当稳态解的所有特征根均具有负实部时,该解为稳定解.利用 Maple 软件,可以求得主振子的响应并进行稳定性分析.

2　数值验证与仿真计算

为验证复变量-平均法推导的近似系统的正确性,使用龙格-库塔法直接对方程(11)进行运算,得

到数值结果后，与近似系统的结果进行对比.并且为获得一般性结论，本章节中所有参数均为无量纲量.系统各项参数如下：$\varepsilon_1=0.05$，$\varepsilon_2=1$，$\gamma_0=0.1$，$\gamma_1=\gamma_2=0.2$，$k_0=225$，$k_{nl1}=200$，$k_{nl2}=20$.简谐激励频率 Ω 变化区间为 10~20.当激励幅值 $A=0.5$ 时，得到连接串联 *NES* 之后主振子的响应.如图 2，使用复变量-平均法以及龙格-库塔法求解出的结果比较接近，尤其在主共振区域十分吻合，这验证了复变量-平均法的适用性与准确性.并且当简谐激励幅值 $A=0.5$ 时，主振子在其固有频率附近的振幅较高，而在连接单个 NES 与串联 NES 之后，主振子在固有频率附近的振幅均有了大幅的降低，如图 3.这说明 NES 对主振子有良好的减振效果，且在较低幅值激励下，串联 NES 和单自由度 NES 具有非常接近的减振能力.

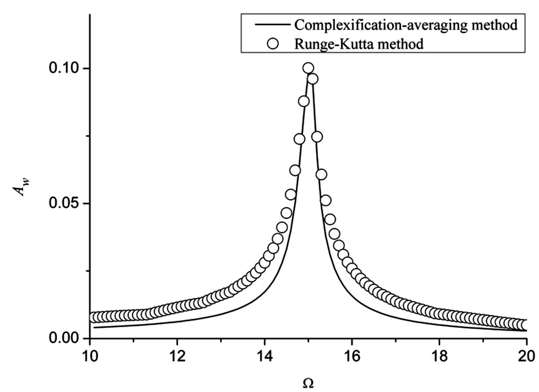

图 2　$A=0.5$ 时主振子的频率响应

Fig.2　Responses of the main oscillator under excitation amplitude $A=0.5$

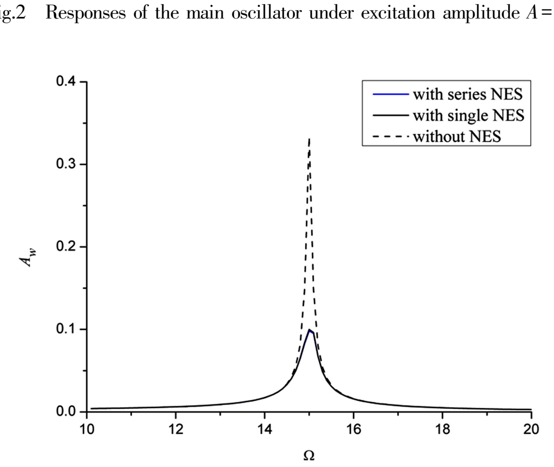

图 3　$A=0.5$ 时连接单/串联 NES 的主振子响应

Fig.3　Responses of the main oscillator attached with SDOF NES and series NES under excitation amplitude $A=0.5$

当激励幅值增大到 $A=1$ 时，如图 4(a)，连接单自由度 NES 的主振子振幅增大，并在 $\Omega=13.7$~14.1 区间出现不稳定解.$A=3$ 时，如图 4(c)，主振子振幅大幅提高，不稳定解的区间范围增大，且在 $\Omega=12.9$~14.0 区间内同时存在两个稳定解，即有高、低两个分支出现.而当激励幅值继续增大到 $A=5$ 时，如图 4(e)，主振子不稳定解的区间范围继续扩大，高分支响应与低分支响应也有合并的趋势，而且主振子的共振振幅较大，连接 NES 后减振效果也并不理想.

高分支响应的出现造成非线性能量阱吸振效能的大幅降低.连接串联 NES 后，在同样激励幅值下重新得到系统响应图.如图 4(b)，4(d)，4(f)，对比连接单个 NES，串联非线性吸振器消除了高分支响应，且在大振幅激励下能将主振子振幅限制在较低范围内，达到良好的减振效果.

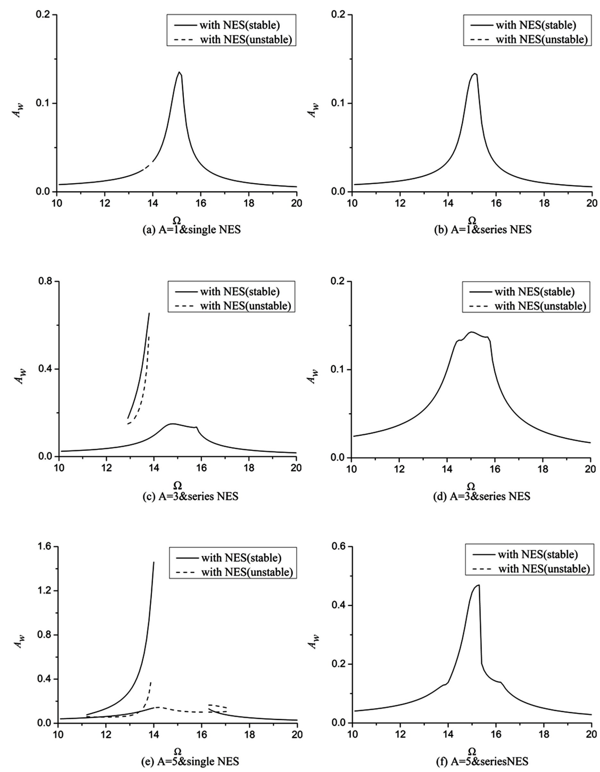

图 4　连接单自由度 NES 与串联 NES 的主振子频率响应

Fig.4　Responses of the main oscillator attached with SDOF NES and series NES

3　串联 NES 参数优化

通过上述分析发现，使用串联 NES 可以在较大激励幅值范围内消除高分支响应并大幅降低主振子振幅，弥补了单自由度 NES 易受大幅激励影响而失效的缺点.为了研究串联 NES 中两级 NES 参数变化对主振子振动抑制的影响，从而对其吸振

效能进行优化,在保持其他参数不变的情况下,分别调整两级 NES 的刚度、阻尼与质量,并在激励幅值 $A=1$ 下得到系统响应如图 5.

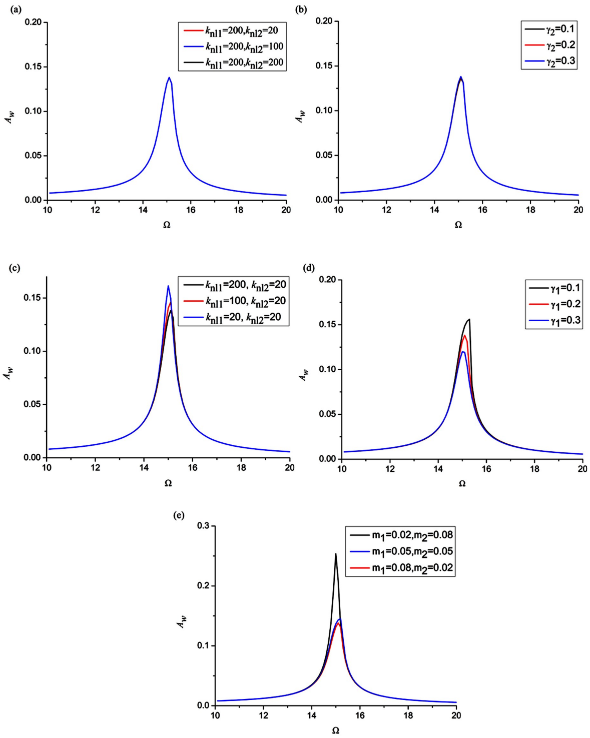

图 5　小幅激励下串联 NES 各参数对主振子振幅的影响

Fig.5　The influences of the parameters of the series NES on the responses of the main oscillator under small excitation amplitude

如图 5(a),5(b),从仿真结果中可以看出,在较低振幅激励下,改变二级吸振器刚度与阻尼,对主振子振幅影响不明显.与其相对的,调整一级 NES 刚度与阻尼,对主振子振动抑制效果则较为明显.如图 5(c),5(d),一级 NES 在 $k_{nl1}=200$,$\gamma_1=0.3$ 时振动抑制效果最佳.并且在保证总质量不变的前提下,合理增大一级 NES 质量,有利于提高吸振效能.

在同样系统参数下,使用大幅激励 $A=5$ 对耦合系统进行激励.研究过程中,除被研究参数的数值变化外,其他所有参数均保持不变.在此条件下,重新调整串联 NES 的各项参数,并获得响应,如图 6~8 所示.

通过对比一级、二级 NES 的三组参数可以发现,在大幅激励下系统出现高分支响应,不同于小幅激励下串联 NES 的表现,此时调整二级 NES 参数对主振子的振动抑制影响较大.首先,减小一、二级 NES 的刚度,可以使串联 NES 在更宽的激励幅值范围内消除高分支,并且相比于调整一级 NES 刚度,降低二级 NES 刚度,对主振子减振效果更为明显;

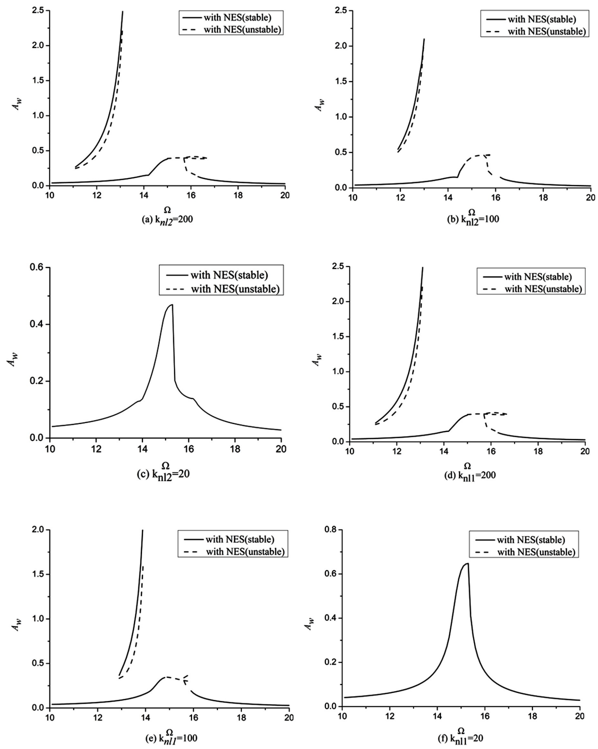

图 6　串联 NES 中两级 NES 的刚度对主振子响应的影响

Fig.6　The influences of two stiffnesses of the series NES on the responses of the main oscillator

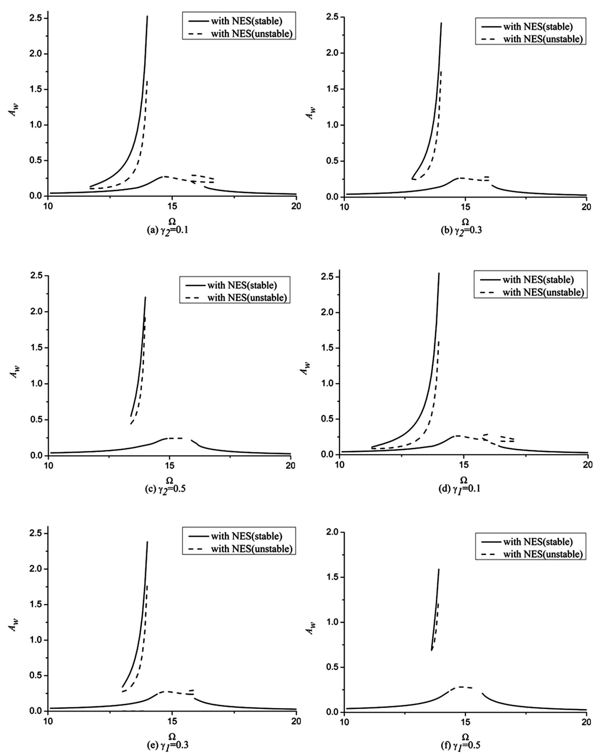

图 7　串联 NES 中两级 NES 的阻尼对主振子响应影响

Fig.7　The influences of two dampings of the series NES on the responses of the main oscillator

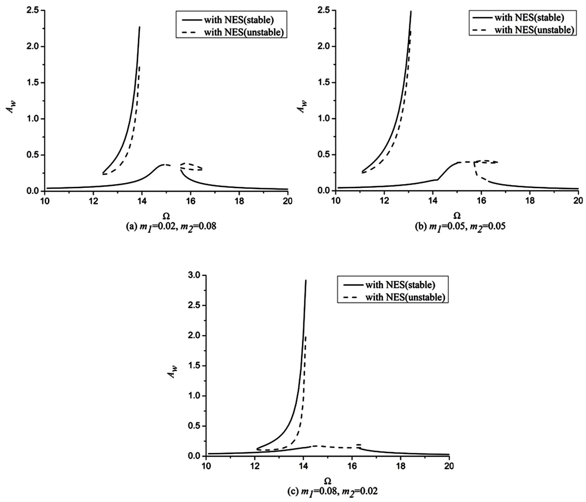

图 8 串联 NES 中两级 NES 的质量对主振子响应影响

Fig.8 The influences of two masses of the series NES on the responses of the main oscillator

其次,通过增大两级 NES 阻尼,也可以在一定范围内减小高分支,降低主振子振幅;最后,在保证串联 NES 总质量不变的原则下,增大二级 NES 的质量,也可以在一定程度上减小主振子的振幅.

4 结论

本文主要研究了串联 NES 的吸振性能,利用复变量-平均法得到了系统的慢变方程,并分析了系统解的稳定性.结果表明,在简谐激励作用下,随着激励振幅的增大,连接单个 NES 的主振子会出现高、低两个分支响应,从而大幅降低 NES 的吸振性能.通过连接串联 NES,可在较大激励幅值范围内消除高分支响应,从而使主振子振幅降低.

另外,在小幅激励环境下,调整二级 NES 参数对主振子影响较小,合理调整一级 NES 参数,即合理增大其阻尼与刚度,在保证总质量不变的前提下,增大一级 NES 质量,可提高串联 NES 的吸振效能;而在大幅激励环境下,调整二级 NES 参数相比于调整一级 NES 参数,对主振子响应的影响更为明显,合理减小二级 NES 刚度,增大二级 NES 阻尼与质量对主振子减振更加有利.在外部激励幅值变化较大,以及载荷方式多样的环境中,串联 NES 具有比单自由度 NES 更佳的应用前景.

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RESEARCH ON HIGHER BRANCH RESPONSE OF SERIES NONLINEAR ENERGY SINK*

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Abstract The vibration absorption efficiencies of a single-degree-of-freedom (SDOF) NES and a 2-degree-of-freedom (2-DOF) series NES were compared,and the suppression of higher branch response by the 2-DOF NES was analyzed. The slow flow equations of the system were derived by complexification-averaging method,and the frequency-response curves were obtained. The response of the system attached with the SDOF NES were compared with that of the system with the series NES, under harmonic excitations with different amplitudes. The results show that the series NES can maintain high vibration absorption efficiency in a large range of excitation amplitudes. Furthermore,the parameter influences of the 2-DOF series NES on the vibration suppression effect were investigated under two excitation amplitudes. The vibration absorption for the host oscillator is mainly influenced by the primary NES under the small-amplitude excitation, but by the secondary NES under the large-amplitude excitation.

Key words nonlinear energy sink(NES), large amplitude excitation, higher branch response, vibration absorption efficiency, series

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