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航空发动机叶片非线性振动分析*

揭晓博　张伟†

(北京工业大学 机电学院, 北京　100022)

摘要　论文研究了航空发动机叶片的非线性振动问题,将叶片简化为旋转圆锥壳,考虑预安装角和预扭转角的影响,利用 Hamilton 原理建立了不同激励作用下的叶片的非线性偏微分运动方程.综合运用 Galerkin 方法和数值方法,对叶片进行了非线性动力学分析,模拟不同转速和激励作用下的叶片运动,得到波形图,讨论了转速和外载荷对旋转叶片非线性动力学特性的影响.

关键词　旋转叶片，　非线性动力学，　圆锥壳，　Hamilton 原理，　Galerkin 方法

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引言

随着现代航空的快速发展,压气机叶片的工作条件更加恶劣,温度和应力水平更高,接近极限.一旦叶片的振动特性发生改变,则会直接影响发动机工作的稳定性.因此,研究压气机叶片的振动情况对保证发动机正常稳定工作具有重要意义.本文主要研究航空发动机压气机旋转叶片的非线性动力学问题.

上世纪七十年代,Rao[1-3] 等将涡轮叶片考虑为有固定转速的悬臂梁,用 Galerkin 法分别研究了带有弯扭耦合和扭转楔形梁的固有频率.1999 年,Bedoor[4] 考虑转子扭转和叶片弯曲变形的耦合作用,运用 Lagrange 法和有限元法推导了固定在轮毂上的弹性叶片的动力学方程. 2001 年, Yoo 和 Chung[5] 用带有预安装角的旋转板来模拟叶片,运用 Kane 法建立运动方程并研究了叶片从启动到稳定状态过程中的动力学响应.2002 年,Chandiramani[6] 等将发动机叶片简化成旋转的盒型梁模型,考虑几何非线性,利用哈密顿原理研究了其振动特性.2004 年,Gurkan[7] 等将叶片简化成含预扭转角的翼型截面梁,利用 Bolotin 方法研究了载荷对其稳定性的影响.2008 年,吴根勇[8] 等基于经典层合板理论,运用 Lagrange 原理和有限元法研究了铺设层、铺设角、轮毂半径等因素对旋转复合材料板非线性振动情况的影响.2011 年,Sinha 和 Turner[9] 考虑弯扭耦合效应,运用薄壳理论建立旋转叶片的模型,将离心力场简化为准静态载荷,研究了旋转叶片的横向偏转.2013 年,Sun 和 Arteaga[10] 等运用厚壳理论,建立预扭的压气机叶片的模型,研究旋转速度和阻尼对叶片振动的影响.2017 年,王晓峰,徐可君和秦海勤[11] 利用有限元分析技术,研究了不同参数对叶片模态的影响.

本文主要研究航空发动机叶片的非线性振动问题.将叶片简化为旋转曲壳,考虑预安装角和预扭转角的影响,利用 Hamilton 原理建立了横向外激励和扭矩联合作用下的叶片的非线性偏微分运动方程.综合运用 Galerkin 方法和数值方法研究外载荷对旋转叶片非线性动力学特性的影响.

1　叶片非线性动力学模型

考虑一个安装在半径为 r_0 的刚性轮毂上的旋转圆锥壳,该壳以转速 $\Omega(t)=\Omega_c+\Omega_v\cos\omega_r t$ 绕转轴旋转,如图 1 所示.坐标系 (x,θ,z) 位于圆锥壳的中面,(u,v,w) 分别表示任意点在 x,θ 和 z 方向上的位移.圆锥壳的几何参数分别为大端半径 r_{root},预安装角 β,预扭转角 Φ,锥顶角为 α,母线长 L,厚度 h,沿母线任意点的半径为 $R=r_{root}-x\tan\psi$.圆锥壳上表面均匀分布横向外激励 $F=F_0+F_1\cos\omega_r t$ 和扭矩 $M=M_0+M_1\cos\omega_r t$,其中外激励 F 和扭矩 M 都由一

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† 通讯作者 E-mail:iron_man1945@163.com

个常量和简谐量构成.

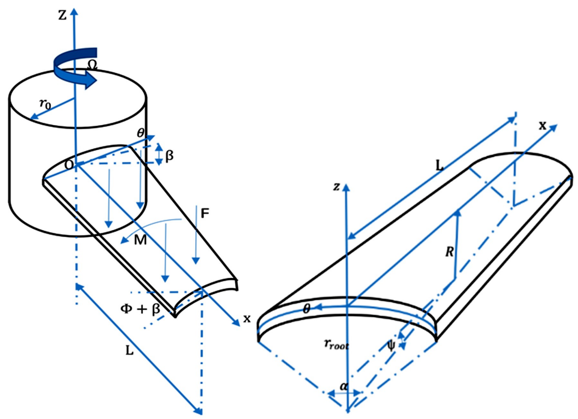


图 1 有预安装角和预扭转角的叶片模型

Fig.1 The model of presetting and pre-twisted blade

根据一阶剪切变形理论,圆锥壳的位移场可以写为:

$$u(x,\theta,z,t)=u_0(x,\theta,t)=z\varphi_x(x,\theta,t) \tag{1a}$$

$$v(x,\theta,z,t)=v_0(x,\theta,t)=z\varphi_\theta(x,\theta,t) \tag{1b}$$

$$w(x,\theta,z,t)=w_0(x,\theta,t) \tag{1c}$$

其中,(u_0,v_0,w_0) 为圆锥壳中面任意一点的位移,φ_x 和 φ_θ 分别表示中面绕 θ 和 x 轴的转角.

非线性位移-应变关系可写为:

$$\begin{bmatrix}\varepsilon_x\\ \varepsilon_\theta\\ \gamma_{x\theta}\end{bmatrix}=\begin{bmatrix}\varepsilon_x^{(0)}\\ \varepsilon_\theta^{(0)}\\ \gamma_{x\theta}^{(0)}\end{bmatrix}+z\begin{bmatrix}\varepsilon_x^{(1)}\\ \varepsilon_\theta^{(1)}\\ \gamma_{x\theta}^{(1)}\end{bmatrix} \tag{2}$$

其中,

$$\begin{bmatrix}\varepsilon_x^{(0)}\\ \varepsilon_\theta^{(0)}\\ \gamma_{x\theta}^{(0)}\end{bmatrix}=\begin{bmatrix}\dfrac{\partial u_0}{\partial x}+\dfrac{1}{2}\left(\dfrac{\partial w_0}{\partial x}\right)^2\\ \dfrac{1}{R\cos\psi}\dfrac{\partial v_0}{\partial\theta}+\dfrac{1}{R}w_0+\dfrac{1}{R}u_0\tan\psi+\dfrac{1}{2}\dfrac{1}{R^2\cos^2\psi}\left(\dfrac{\partial w_0}{\partial\theta}\right)^2\\ \dfrac{1}{R\cos}\dfrac{\partial u_0}{\partial\theta}-\dfrac{1}{R}v_0\tan\psi+\dfrac{\partial v_0}{\partial x}+\dfrac{1}{R\cos\psi}\dfrac{\partial w_0}{\partial x}\dfrac{\partial w_0}{\partial\theta}\end{bmatrix} \tag{3}$$

$$\begin{bmatrix}\varepsilon_x^{(1)}\\ \varepsilon_\theta^{(1)}\\ \gamma_{x\theta}^{(1)}\end{bmatrix}=\begin{bmatrix}\dfrac{\partial\varphi_x}{\partial x}\\ \dfrac{1}{R\cos\psi}\dfrac{\partial\varphi_\theta}{\partial\theta}+\dfrac{1}{R}\varphi_x\tan\psi\\ \dfrac{1}{R\cos\psi}\dfrac{\partial\varphi_x}{\partial\theta}-\dfrac{1}{R}\varphi_\theta\tan\psi+\dfrac{\partial\varphi_\theta}{\partial x}\end{bmatrix} \tag{4}$$

$$\begin{bmatrix}\gamma_{\theta z}\\ \gamma_{xz}\end{bmatrix}=\begin{bmatrix}\varphi_\theta+\dfrac{1}{R\cos\psi}\dfrac{\partial w_0}{\partial\theta}-\dfrac{1}{R}v_0\\ \dfrac{\partial w_0}{\partial x}+\varphi_x\end{bmatrix} \tag{5}$$

壳的应力-应变关系可表示为

$$\begin{bmatrix}\sigma_x\\ \sigma_\theta\\ \sigma_{x\theta}\\ \sigma_{\theta z}\\ \sigma_{xz}\end{bmatrix}=\begin{bmatrix}Q_{11} & Q_{12} & & & \\ Q_{12} & Q_{22} & & & \\ & & Q_{66} & & \\ & & & Q_{44} & \\ & & & & Q_{55}\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_\theta\\ \gamma_{x\theta}\\ \gamma_{\theta z}\\ \gamma_{xz}\end{bmatrix} \tag{6}$$

其中,$Q_{mn}(m,n=1,2,4,5,6)$ 分别为:

$$Q_{11}=Q_{22}=\frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$$

$$Q_{12}=\frac{\mathrm{E}\nu}{(1+\nu)(1-2\nu)}$$

$$Q_{44}=Q_{55}=Q_{66}=\frac{E}{2(1+\nu)} \tag{7}$$

这里 E 表示杨氏模量,ν 表示泊松比.

根据 Hamilton 原理,得到系统的非线性动力学偏微分方程为:

$$\begin{aligned}&A_{11}\frac{\partial^2u_0}{\partial x^2}+A_{66}\frac{1}{R^2\cos^2\psi}\frac{\partial^2u_0}{\partial\theta^2}+(A_{12}-A_{22})\cdot\\ &\tan^2\psi\frac{1}{R^2}u_0+(A_{12}-A_{22}-A_{66})\frac{\tan\psi}{\cos\psi}\frac{1}{R^2}\frac{\partial v_0}{\partial\theta}+\\ &(A_{12}+A_{66})\frac{1}{R\cos\psi}\frac{\partial^2v_0}{\partial x\partial\theta}+\left(A_{12}-\frac{A_{22}}{2}\right)\frac{\tan\psi}{\cos^2\psi}\cdot\\ &\frac{1}{R^3}\left(\frac{\partial w_0}{\partial\theta}\right)^2+A_{12}\frac{1}{R}\frac{\partial w_0}{\partial x}-\frac{A_{12}}{2}\tan\psi\frac{1}{R}\left(\frac{\partial w_0}{\partial x}\right)^2+\\ &(A_{12}-A_{22})\tan\psi\frac{1}{R^2}w_0+A_{11}\frac{\partial w_0}{\partial x}\frac{\partial^2w_0}{\partial x^2}+\\ &A_{66}\frac{1}{R^2\cos^2\psi}\frac{\partial w_0}{\partial x}\frac{\partial^2w_0}{\partial\theta^2}+(A_{12}+A_{66})\sec^2\psi\cdot\\ &\frac{1}{R^2}\frac{\partial w_0}{\partial\theta}\frac{\partial^2w_0}{\partial x\partial\theta}+B_{11}\frac{\partial^2\varphi_x}{\partial x^2}+B_{66}\frac{1}{R^2\cos^2\psi}\frac{\partial^2\varphi_x}{\partial\theta^2}+\\ &(B_{12}-B_{22})\tan^2\psi\frac{1}{R^2}\varphi_x+(B_{12}-B_{22}-B_{66})\cdot\\ &\frac{\tan\psi}{\cos\psi}\frac{1}{R^2}\frac{\partial\varphi_\theta}{\partial\theta}+(B_{12}+B_{66})\frac{1}{R\cos\psi}\frac{\partial^2\varphi_\theta}{\partial x\partial\theta}\\ &=I_0\ddot{u}_0-2\Omega^2I_0u_0+2\Omega(I_0+I_1)(R+1)\cdot\\ &\quad\cos[\theta-\beta(x)]\dot{v}_0+\dot{\Omega}(I_0+I_1)(R+1)\cdot\\ &\quad\cos[\theta-\beta(x)]v_0+2\Omega(I_0+I_1)(R-1)\cdot\\ &\quad\sin[\theta-\beta(x)]\dot{w}_0+\dot{\Omega}(I_0+I_1)(R-1)\cdot\\ &\quad\sin[\theta-\beta(x)]w_0+I_1\ddot{\varphi}_x-\Omega^2I_1\varphi_x+\\ &\quad2\Omega(I_1+I_2)(R+1)\cos[\theta-\beta(x)]\varphi_\theta+\\ &\quad\dot{\Omega}I_2\varphi_\theta+(-2\Omega^2I_0(r_0+x)+(I_0R+I_1)\cdot\end{aligned}$$

$$\theta\dot{\Omega}\cos[\theta-\beta(x)]+(I_1R+I_2)\dot{\Omega}\sin[\theta-\beta(x)])\tag{8a}$$

$$(A_{22}+2A_{66})\frac{\tan\psi}{R^2\cos\psi}\frac{\partial u_0}{\partial\theta}+(A_{12}+A_{66})\cdot$$

$$\frac{1}{R\cos\psi}\frac{\partial^2 u_0}{\partial x\partial\theta}+A_{66}\frac{\partial^2 v_0}{\partial x^2}+A_{22}\frac{1}{R^2\cos^2\psi}\frac{\partial^2 v_0}{\partial\theta^2}-$$

$$(A_{55}+2A_{66}\tan^2\psi)\frac{1}{R^2}v_0+(A_{22}+A_{55})\cdot$$

$$\frac{1}{R^2\cos\psi}\frac{\partial w_0}{\partial\theta}+A_{66}\frac{\tan\psi}{\cos\psi}\frac{1}{R^2}\frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial\theta}+$$

$$(A_{12}+A_{66})\frac{1}{R\cos\psi}\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial x\partial\theta}+A_{66}\frac{1}{R\cos\psi}\cdot$$

$$\frac{\partial w_0}{\partial\theta}\frac{\partial^2 w_0}{\partial x^2}+A_{22}\frac{1}{R^3\cos^3\psi}\frac{\partial w_0}{\partial\theta}\frac{\partial^2 w_0}{\partial\theta^2}+$$

$$(B_{22}+2B_{66})\frac{\tan\psi}{\cos\psi}\frac{1}{R^2}2\frac{\partial\varphi_x}{\partial\theta}+(B_{12}+B_{66})\cdot$$

$$\frac{1}{R\cos\psi}\frac{\partial^2\varphi_x}{\partial x\partial\theta}+B_{66}\frac{\partial^2\varphi_\theta}{\partial x^2}+B_{22}\frac{1}{R^2\cos^2\psi}\cdot$$

$$\frac{\partial^2\varphi_\theta}{\partial\theta^2}+\left(A_{55}\frac{1}{R}-2B_{66}\tan^2\psi\frac{1}{R^2}\right)\varphi_\theta$$

$$=-2\Omega\cos[\theta-\beta(x)](I_0R+I_1)\dot{u}_0-$$

$$\dot{\Omega}\cos[\theta-\beta(x)](I_0R+I_1)u_0+I_0\ddot{v}_0-$$

$$\Omega^2(1+\cos[2(\theta-\beta(x))])(R^2I_0+2RI_1+I_2)v_0-$$

$$\Omega^2\sin[2(\theta-\beta(x))](R^2I_0+2RI_1+I_2)w_0-$$

$$2\Omega\cos[\theta-\beta(x)](RI_1+I_2)\varphi_x-$$

$$I_2\cos[\theta-\beta(x)]\dot{\Omega}\varphi_x+I_1\ddot{\varphi}_\theta-$$

$$\Omega^2(1+\cos[2(\theta-\beta(x))])\left(\frac{1}{2}R^2I_1+I_3\right)\varphi_\theta-$$

$$(R^2I_0+2RI_1+I_2)(1+\cos[2(\theta-\beta(x))])\Omega^2\theta-$$

$$(RI_0+I_1)\dot{\Omega}\cos[\theta-\beta(x)](r_0+x)-$$

$$(R^2I_1+2RI_2+I_3)\Omega^2\sin[2(\theta-\beta(x))]\tag{8b}$$

$$A_{11}\frac{\partial^2 u_0}{\partial x^2}\frac{\partial w_0}{\partial x}+A_{66}\frac{1}{R^2\cos^2\psi}\frac{\partial^2 u_0}{\partial\theta^2}\frac{\partial w_0}{\partial x}+$$

$$(A_{12}+A_{66})\frac{1}{R^2\cos^2\psi}\frac{\partial^2 u_0}{\partial x\partial\theta}\frac{\partial w_0}{\partial\theta}-A_{12}\frac{1}{R}\frac{\partial u_0}{\partial x}+$$

$$A_{12}\tan\psi\frac{1}{R}\frac{\partial u_0}{\partial x}\frac{\partial w_0}{\partial x}+(A_{22}+A_{66})\frac{\tan\psi}{R^3\cos^2\psi}\cdot$$

$$\frac{\partial u_0}{\partial\theta}\frac{\partial w_0}{\partial\theta}-A_{22}\frac{\tan\psi}{R^2}u_0+A_{12}\frac{\tan^2\psi}{R^2}u_0\frac{\partial w_0}{\partial x}+$$

$$A_{66}\frac{1}{R\cos\psi}\frac{\partial^2 v_0}{\partial x^2}\frac{\partial w_0}{\partial\theta}+A_{22}\frac{1}{R^3\cos^3\psi}\cdot$$

$$\frac{\partial^2 v_0}{\partial\theta^2}\frac{\partial w_0}{\partial\theta}+(A_{12}+A_{66})\frac{1}{R\cos\psi}\frac{\partial^2 v_0}{\partial x\partial\theta}\frac{\partial w_0}{\partial x}-$$

$$A_{66}\frac{\tan\psi}{R^2\cos\psi}\frac{\partial v_0}{\partial x}\frac{\partial w_0}{\partial\theta}-(A_{22}+A_{55})\frac{1}{R^2\cos\psi}\frac{\partial v_0}{\partial\theta}+$$

$$(A_{12}-A_{66})\frac{\tan\psi}{\cos\psi}\frac{1}{R^2}\frac{\partial v_0}{\partial\theta}\frac{\partial w_0}{\partial x}-$$

$$A_{66}\frac{\tan^2\psi}{R^3\cos\psi}v_0\frac{\partial w_0}{\partial\theta}+A_{44}\frac{\partial^2 w_0}{\partial x^2}+A_{11}\frac{\partial^2 w_0}{\partial x^2}\left(\frac{\partial w_0}{\partial x}\right)^2+$$

$$A_{66}\frac{1}{R^2\cos^2\psi}\frac{\partial^2 w_0}{\partial x^2}\left(\frac{\partial w_0}{\partial\theta}\right)^2+A_{55}\frac{1}{R^2\cos^2\psi}\frac{\partial^2 w_0}{\partial\theta^2}+$$

$$A_{66}\frac{1}{R^2\cos^2\psi}\frac{\partial^2 w_0}{\partial\theta^2}\left(\frac{\partial w_0}{\partial x}\right)^2+A_{22}\frac{1}{R^4\cos\psi^4}\frac{\partial^2 w_0}{\partial\theta^2}\left(\frac{\partial w_0}{\partial\theta}\right)^2+$$

$$2(A_{12}+A_{66})\frac{1}{R^2\cos^2\psi}\frac{\partial^2 w_0}{\partial x\partial\theta}\frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial\theta}+$$

$$(A_{12}+A_{66})\frac{\tan\psi}{R^3\cos^2\psi}\frac{\partial w_0}{\partial x}\left(\frac{\partial w_0}{\partial\theta}\right)^2+$$

$$\frac{A_{12}}{2}\frac{1}{R}\left(\frac{\partial w_0}{\partial x}\right)^2+\frac{A_{22}}{2}\frac{1}{R^3\cos^2\psi}\left(\frac{\partial w_0}{\partial\theta}\right)^2+$$

$$A_{12}\frac{\tan\psi}{R^2}w_0\frac{\partial w_0}{\partial x}-A_{22}\frac{1}{R^2}w_0+B_{11}\frac{\partial^2\varphi_x}{\partial x^2}\frac{\partial w_0}{\partial x}+$$

$$B_{66}\frac{1}{R^2\cos^2\psi}\frac{\partial^2\varphi_x}{\partial\theta^2}\frac{\partial w_0}{\partial x}+(B_{12}+B_{66})\frac{1}{R^2\cos^2\psi}\frac{\partial^2\varphi_x}{\partial x\partial\theta}\frac{\partial w_0}{\partial\theta}+$$

$$\left(A_{44}-B_{12}\frac{1}{R}\right)\frac{\partial\varphi_x}{\partial x}+B_{12}\tan\psi\frac{1}{R}\frac{\partial\varphi_x}{\partial x}\frac{\partial w_0}{\partial x}+$$

$$(B_{22}+B_{66})\frac{\tan\psi}{R^3\cos^2\psi}\frac{\partial\varphi_x}{\partial\theta}\frac{\partial w_0}{\partial\theta}+B_{12}\tan\psi^2\frac{1}{R^2}\varphi_x\frac{\partial w_0}{\partial x}-$$

$$B_{22}\frac{\tan\psi}{R^2}\varphi_x+B_{66}\frac{1}{R\cos\psi}\frac{\partial^2\varphi_\theta}{\partial x^2}\frac{\partial w_0}{\partial\theta}+$$

$$B_{22}\frac{1}{R^3\cos^3\psi}\frac{\partial^2\varphi_\theta}{\partial\theta^2}\frac{\partial w_0}{\partial\theta}+(B_{12}+B_{66})\frac{1}{R\cos\psi}\frac{\partial^2\varphi_\theta}{\partial x\partial\theta}\frac{\partial w_0}{\partial x}-$$

$$B_{66}\frac{\tan\psi}{R^2\cos\psi}\frac{\partial\varphi_\theta}{\partial x}\frac{\partial w_0}{\partial\theta}+\left(A_{55}-B_{22}\frac{1}{R}\right)\frac{1}{R\cos\psi}\frac{\partial\varphi_\theta}{\partial\theta}+$$

$$(B_{12}-B_{66})\frac{\tan\psi}{\cos\psi}\frac{1}{R^2}\frac{\partial\varphi_\theta}{\partial\theta}\frac{\partial w_0}{\partial x}-B_{66}\frac{\tan^2\psi}{R^3\cos\psi}\varphi_\theta\frac{\partial w_0}{\partial\theta}$$

$$=I_0\ddot{w}_0-(R^2I_0+2RI_1+2I_2)\Omega^2w_0\cdot$$

$$(1-\cos[2(\theta-\beta(x))])-2(RI_0+I_1)\Omega\cdot$$

$$\sin[\theta-\beta(x)]\dot{u}_0-(RI_0+I_1)\dot{\Omega}\sin[\theta-\beta(x)]u_0-$$

$$(R^2I_0+2RI_1+I_2)\Omega^2\sin[2(\theta-\beta(x))]v_0-$$

$$2(RI_1+I_2)\Omega\sin[\theta-\beta(x)]\varphi_x-I_2\dot{\Omega}\cdot$$

$$\sin[\theta-\beta(x)]\varphi_x-\left(\frac{1}{2}R^2I_1+2RI_2+I_3\right)\cdot$$

$$\Omega^2\sin[2(\theta-\beta(x))]\varphi_\theta-I_0(R\dot{\Omega}(r_0+x)\cdot \sin[\theta-\beta(x)]+R^2\Omega^2\sin[2(\theta-\beta(x))]\theta)-I_1(2R\Omega^2\sin[2(\theta-\beta(x))]\theta+\dot{\Omega}(r_0+x)\cdot \sin[\theta-\beta(x)]+R^2\Omega^2(1-\cos[2(\theta-\beta(x))])-I_2\Omega^2(\sin[2(\theta-\beta(x))]\theta+(1-\cos[2(\theta-\beta(x))])\cdot R)-I_3\Omega^2(1-\cos[2(\theta-\beta(x))])+F-\kappa\dot{w}_0 \tag{8c}$$

$$B_{11}\frac{\partial^2 u_0}{\partial x^2}+B_{66}\frac{1}{R^2\cos^2\psi}\frac{\partial^2 u_0}{\partial\theta^2}+(B_{12}-B_{22})\cdot \tan^2\psi\frac{1}{R^2}u_0+(B_{12}-B_{22}-B_{66})\frac{\tan\psi}{\cos\psi}\frac{1}{R^2}\frac{\partial v_0}{\partial\theta}+(B_{12}+B_{66})\frac{1}{R\cos\psi}\frac{\partial^2 v_0}{\partial x\partial\theta}+(B_{12}-\frac{B_{22}}{2})\cdot \frac{\tan\psi}{\cos^2\psi}\frac{1}{R^3}\left(\frac{\partial w_0}{\partial\theta}\right)^2+(B_{12}\frac{1}{R}-A_{44})\frac{\partial w_0}{\partial x}-\frac{B_{12}}{2}\tan\psi\frac{1}{R}\left(\frac{\partial w_0}{\partial x}\right)^2+(B_{12}-B_{22})\tan\psi\frac{1}{R^2}w_0+B_{11}\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial x^2}+B_{66}\frac{1}{R^2\cos^2\psi}\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial\theta^2}+(B_{12}+B_{66})\sec^2\psi\frac{1}{R^2}\frac{\partial w_0}{\partial\theta}\frac{\partial^2 w_0}{\partial x\partial\theta}+D_{11}\frac{\partial^2\varphi_x}{\partial x^2}+D_{66}\frac{1}{R^2\cos^2\psi}\frac{\partial^2\varphi_x}{\partial\theta^2}+((D_{12}-D_{22})\tan^2\psi\cdot \frac{1}{R^2}-A_{44})\varphi_x+(D_{12}-D_{22}-D_{66})\frac{\tan\psi}{\cos\psi}\frac{1}{R^2}\cdot \frac{\partial\varphi_\theta}{\partial\theta}+(D_{12}+D_{66})\frac{1}{R\cos\psi}\frac{\partial^2\varphi_\theta}{\partial x\partial\theta}$$
$$=I_1\ddot{u}_0-2\Omega^2 I_1 u_0+2\Omega(I_1+I_2)(R+1)\cdot \cos[\theta-\beta(x)]\dot{v}_0+\dot{\Omega}(I_1+I_2)(R+1)\cdot \cos[\theta-\beta(x)]v_0+2\Omega(I_1+I_2)(R-1)\cdot \sin[\theta-\beta(x)]\dot{w}_0+\dot{\Omega}(I_1+I_2)(R-1)\cdot \sin[\theta-\beta(x)]w_0+I_2\ddot{\varphi}_x-\Omega^2 I_2\varphi_x+2\Omega\cdot (I_2+I_3)(R+1)\cos[\theta-\beta(x)]\dot{\varphi}_\theta+\dot{\Omega}I_3\varphi_\theta+(-2\Omega^2 I_1(r_0+x)+(I_1R+I_2)\cos[\theta-\beta(x)]\theta\dot{\Omega}+(I_2R+I_3)\dot{\Omega}\sin[\theta-\beta(x)]) \tag{8d}$$

$$(B_{22}+2B_{66})\frac{\tan\psi}{R^2\cos\psi}\frac{\partial u_0}{\partial\theta}+(B_{12}+B_{66})\cdot \frac{1}{R\cos\psi}\frac{\partial^2 u_0}{\partial x\partial\theta}+B_{66}\frac{\partial^2 v_0}{\partial x^2}+B_{22}\frac{1}{R^2\cos^2\psi}\frac{\partial^2 v_0}{\partial\theta^2}+(A_{55}-2B_{66}\frac{1}{R}\tan^2\psi)\frac{1}{R}v_0+(B_{22}\frac{1}{R}-A_{55})\frac{1}{R\cos\psi}\frac{\partial w_0}{\partial\theta}+B_{66}\frac{\tan\psi}{\cos\psi}\frac{1}{R^2}\frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial\theta}+(B_{12}+B_{66})\frac{1}{R\cos\psi}\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial x\partial\theta}+B_{66}\frac{1}{R\cos\psi}\frac{\partial w_0}{\partial\theta}\frac{\partial^2 w_0}{\partial x^2}+B_{22}\frac{1}{R^3\cos^3\psi}\frac{\partial w_0}{\partial\theta}\frac{\partial^2 w_0}{\partial\theta^2}+(D_{22}+2D_{66})\frac{\tan\psi}{\cos\psi}\frac{1}{R^2}\frac{\partial\varphi_x}{\partial\theta}+(D_{12}+D_{66})\frac{1}{R\cos\psi}\frac{\partial^2\varphi_x}{\partial x\partial\theta}+D_{66}\frac{\partial^2\varphi_\theta}{\partial x^2}+D_{22}\frac{1}{R^2\cos^2\psi}\frac{\partial^2\varphi_\theta}{\partial\theta^2}-(A_{55}+2D_{66}\tan\psi^2\frac{1}{R^2})\varphi_\theta$$
$$=-2\Omega\cos[\theta-\beta(x)](I_1R+I_2)\dot{u}_0-\dot{\Omega}\cdot \cos[\theta-\beta(x)](I_1R+I_2)u_0+I_1\ddot{v}_0-\Omega^2\cdot (1+\cos[2(\theta-\beta(x))])(R^2I_1+2\mathrm{RI}_2+I_3)v_0-\Omega^2\sin[2(\theta-\beta(x))](R^2I_1+2RI_2+I_3)w_0-2\Omega\cos[\theta-\beta(x)](\mathrm{RI}_2+I_3)\dot{\varphi}_x-I_3\cos[\theta-\beta(x)]\dot{\Omega}\varphi_x+I_2\ddot{\varphi}_\theta-\left(\frac{1}{2}R^2I_2+I_4\right)\cdot (1+\cos[2(\theta-\beta(x))])\Omega^2\varphi_\theta-\Omega^2\theta(1+\cos[2(\theta-\beta(x))])(R^2I_1+2RI_2+I_3)-(\mathrm{RI}_1+I_2)\dot{\Omega}\cos[\theta-\beta(x)](r_0+x)-\Omega^2(R^2I_2+2\,\mathrm{RI}_3+I_4)\sin[2(\theta-\beta(x))]-M+\gamma\dot{\varphi}_\theta \tag{8e}$$

其中 $R=r_{root}-x\tan\psi$.

边界条件为:

$x=0$,

$$u_0=v_0=w_0=\varphi_x=\varphi_\theta=0 \tag{9a}$$

$x=L$,

$$N_{xx}=N_{\theta\theta}=N_{x\theta}=M_{xx}=M_{\theta\theta}=M_{x\theta}=Q_x=Q_\theta=0 \tag{9b}$$

$\theta=-\frac{\pi}{2}$和$\frac{\pi}{2}$,

$$N_{xx}=N_{\theta\theta}=N_{x\theta}=M_{xx}=M_{\theta\theta}=M_{x\theta}=Q_x=Q_\theta=0 \tag{9c}$$

2 Galerkin 离散

对方程(8)进行无量纲化,然后应用 Galerkin 方法将偏微分形式的非线性方程离散为常微分形式的非线性动力学方程.本文选取了系统前两阶振动模态进行二阶 Galerkin 离散,在满足位移边界条件的情况下选取振型函数为:

$$u_0(x,\theta,t)=u_1(t)\cos\left(\frac{\pi}{L}x\right)\cos(3\theta-3x)+$$

$$u_2(t)\cos\left(\frac{3\pi}{L}x\right)\cos(\theta-x) \tag{10a}$$

$$v_0(x,\theta,t)=v_1(t)\sin\left(\frac{\pi}{L}x\right)\sin(3\theta-3x)+$$
$$v_2(t)\sin\left(\frac{3\pi}{L}x\right)\sin(\theta-x) \tag{10b}$$

$$w_0(x,\theta,t)=w_1(t)\sin\left(\frac{\pi}{L}x\right)\cos(3\theta-3x)+$$
$$w_2(t)\sin\left(\frac{3\pi}{L}x\right)\cos(\theta-x) \tag{10c}$$

$$\varphi_x(x,\theta,t)=\varphi_{x_1}(t)\cos\left(\frac{\pi}{L}x\right)\cos(3\theta-3x)+$$
$$\varphi_{x_2}(t)\cos\left(\frac{3\pi}{L}x\right)\cos(\theta-x) \tag{10d}$$

$$\varphi_\theta(x,\theta,t)=\varphi_{\theta_1}(t)\sin\left(\frac{\pi}{L}x\right)\sin(3\theta-3x)+$$
$$\varphi_{\theta_2}(t)\sin\left(\frac{3\pi}{L}x\right)\sin(\theta-x) \tag{10e}$$

利用 Galerkin 法进行离散,得到常微分方程:

$$\begin{aligned}&\ddot{w}_1+\mu_1\dot{w}_1+\omega_1^2w_1+\alpha_1\Omega_v\cos\omega_rtw_1+\\&\alpha_2\Omega_v{}^2(\cos\omega_rt)^2w_1+\alpha_3w_1^2+\alpha_4w_2^2+\alpha_5w_1w_2+\\&\alpha_6w_1^2w_2+\alpha_7w_1^3+\alpha_8w_2^3+\alpha_9\varphi_{\theta_1}+\alpha_{10}\Omega_v\cos\omega_rt\varphi_{\theta_1}+\\&\alpha_{11}w_1\varphi_{\theta_2}+\alpha_{12}\Omega_v{}^2(\cos\omega_rt)^2\varphi_{\theta_1}+\alpha_{13}w_1\varphi_{\theta_1}+\\&\alpha_{14}w_2\varphi_{\theta_1}+\alpha_{15}w_2\varphi_{\theta_2}+\alpha_{16}+\alpha_{17}\Omega_v\sin\omega_rt+\\&\alpha_{18}\Omega_v\cos\omega_rt+\alpha_{19}\Omega_v{}^2(\cos\omega_rt)^2=\alpha_{20}F_1\cos\omega_rt\end{aligned} \tag{11a}$$

$$\begin{aligned}&\ddot{w}_2+\mu_2\dot{w}_2+\omega_2^2w_2+\beta_1\Omega_v\cos\omega_rtw_2+\\&\beta_2\Omega_v{}^2(\cos\omega_rt)^2w_2+\beta_3w_1^2+\beta_4w_2^2+\beta_5w_1w_2+\\&\beta_6w_1^2w_2+\beta_7w_1^3+\beta_8w_2^3+\beta_9\varphi_{\theta_2}+\beta_{10}\Omega_v\cos\omega_rt\varphi_{\theta_2}+\\&\beta_{11}w_1\varphi_{\theta_2}+\beta_{12}\Omega_v{}^2(\cos\omega_rt)^2\varphi_{\theta_2}+\beta_{13}w_1\varphi_{\theta_1}+\\&\beta_{14}w_2\varphi_{\theta_1}+\beta_{15}w_2\varphi_{\theta_2}+\beta_{16}+\beta_{17}\Omega_v\sin\omega_rt+\\&\beta_{18}\Omega_v\cos\omega_rt+\beta_{19}\Omega_v^2(\cos\omega_rt)^2=\beta_{20}F_1\cos\omega_rt\end{aligned} \tag{11b}$$

$$\begin{aligned}&\ddot{\varphi}_{\theta_1}+\mu_3\dot{\varphi}_{\theta_1}+\omega_3^2\varphi_{\theta_1}+\gamma_1\Omega_v\cos\omega_rt\varphi_{\theta_1}+\\&\gamma_2\Omega_v{}^2(\cos\omega_rt)^2\varphi_{\theta_1}+\gamma_3w_1+\gamma_4\Omega_v\cos\omega_rtw_1+\\&\gamma_5\Omega_v{}^2(\cos\omega_rt)^2w_1+\gamma_6w_1^2+\gamma_7w_1w_2+\gamma_8w_2^2+\\&\gamma_9+\gamma_{10}\Omega_v\sin\omega_rt+\gamma_{11}\Omega_v\cos\omega_rt+\\&\gamma_{12}\Omega_v^2(\cos\omega_rt)^2=\gamma_{13}M_1\cos\omega_rt\end{aligned} \tag{11c}$$

$$\begin{aligned}&\ddot{\varphi}_{\theta_2}+\mu_4\dot{\varphi}_{\theta_2}+\omega_4^2\varphi_{\theta_2}+\eta_1\Omega_v\cos\omega_rt\varphi_{\theta_2}+\\&\eta_2\Omega_v{}^2(\cos\omega_rt)^2\varphi_{\theta_2}+\eta_3w_1+\eta_4\Omega_v\cos\omega_rtw_1+\\&\eta_5\Omega_v{}^2(\cos\omega_rt)^2w_1+\eta_6w_1^2+\eta_7w_1w_2+\eta_8w_2^2+\\&\eta_9+\eta_{10}\Omega_v\sin\omega_rt+\eta_{11}\Omega_v\cos\omega_rt+\\&\eta_{12}\Omega_v^2(\cos\omega_rt)^2=\eta_{13}M_1\cos\omega_rt\end{aligned} \tag{11d}$$

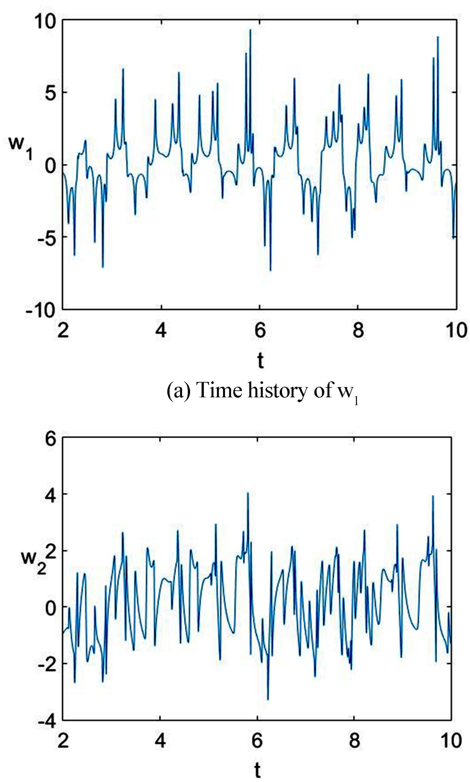
3　算例分析

考虑具有如下几何参数和材料属性的旋转圆锥壳 $L=0.18\text{m}$, $r_{root}=0.08\text{m}$, $\psi=\frac{\pi}{4}$, $\nu=0.33$, $E=102.04\text{ GPa}$, $h_0=0.006\text{m}$, $\omega_r=150$, $\alpha=\frac{\pi}{3}$, $\rho=4450\text{kg/m}^3$, $\kappa=300\text{Ns/m}$, $F_0=1.5\times10^6\text{N/m}^2$, $M_0=1.2\times10^6\text{N/m}$, $\Omega_c=2500\text{r/min}$, $\beta=\frac{\pi}{6}$, $\gamma=300\text{Ns/m}$. 分析系统的周期运动,通过 Runge-Kutta 法,得到时间历程图.

3.1　扰动转速对叶片非线性动力学现象的影响

为了研究扰动转速对系统非线性动力学特性的影响,以 Ω_v 为控制参数,研究其对系统非线性动力学响应的影响.

选定参数 $\Omega_v=0.21$,画出系统的波形图进行研究.(a)(b)分别是 (t,w_1)、(t,w_2) 平面上的波形图.



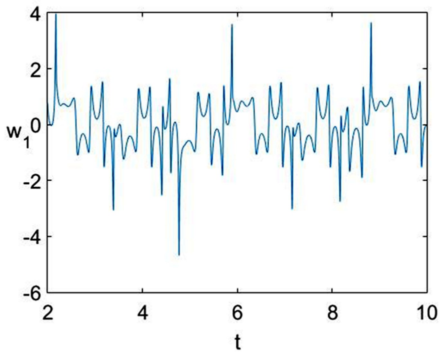
(a) Time history of w_1

(b) Time history of w_2

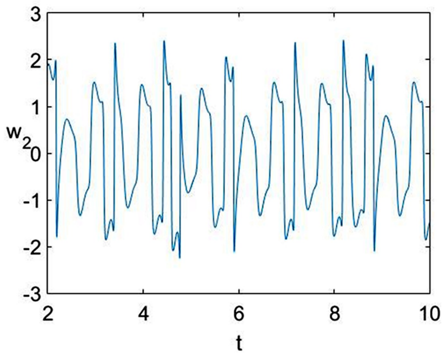
图 2　叶片振动波形图

Fig.2　The vibration waveform of the blade

选定参数 $\Omega_v=0.3$,画出系统的波形图进行研究.(a)(b)分别是 (t,w_1)、(t,w_2) 平面上的波形图.



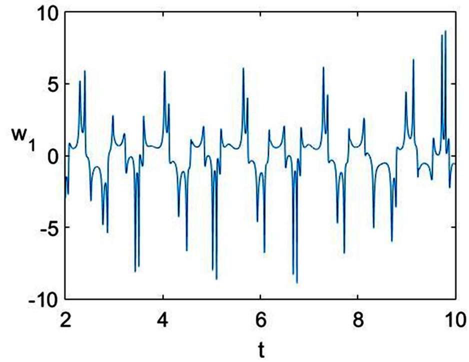
(a) Time history of w_1



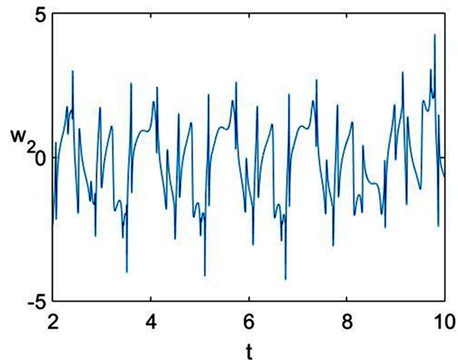
(b) Time history of w_2

图 3 叶片振动波形图

Fig.3 The vibration waveform of the blade



(a) Time history of w_1



(b) Time history of w_2

图 4 叶片振动波形图

Fig.4 The vibration waveform of the blade

通过分析,可以看出扰动速度对圆锥壳的影响.首先可以观察到,悬臂圆锥壳存在稳态振动.其次,当圆锥壳的转速增加的时候,如图 3 可以看到,伴随旋转速度的增加,w_1 和 w_2 的幅值有减小的趋势,该现象可解释为叶片在旋转过程中结构的刚度增加.

3.2 扰动力对叶片非线性动力学现象的影响

为了研究扰动力对系统非线性动力学特性的影响,以 F_1 为控制参数,研究其对系统非线性动力学响应的影响.

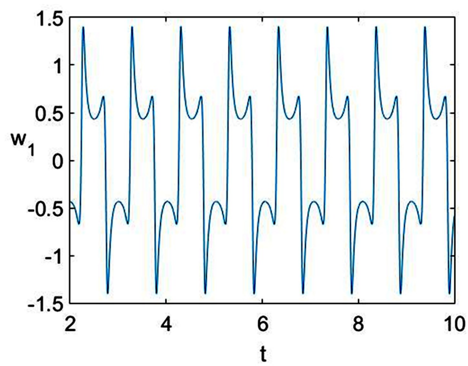
选定参数 $F_1=0.5$,画出系统的波形图进行研究.(a)(b)分别是(t,w_1)、(t,w_2)平面上的波形图.

选定参数 $F_1=0.83$,画出系统的波形图进行研究.(a)(b)分别是(t,w_1)、(t,w_2)平面上的波形图.

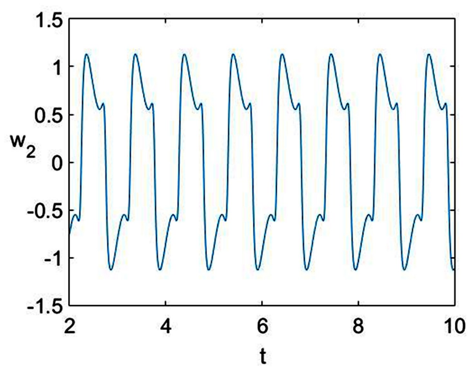
旋转悬臂圆锥壳上的横向外力增加后,w_1 和 w_2 的幅值增大.

3.3 扰动扭矩对叶片非线性动力学现象的影响

为了研究扰动扭矩对系统非线性动力学特性的影响,以 M_1 为控制参数,研究其对系统非线性动力学响应的影响.



(a) Time history of w_1

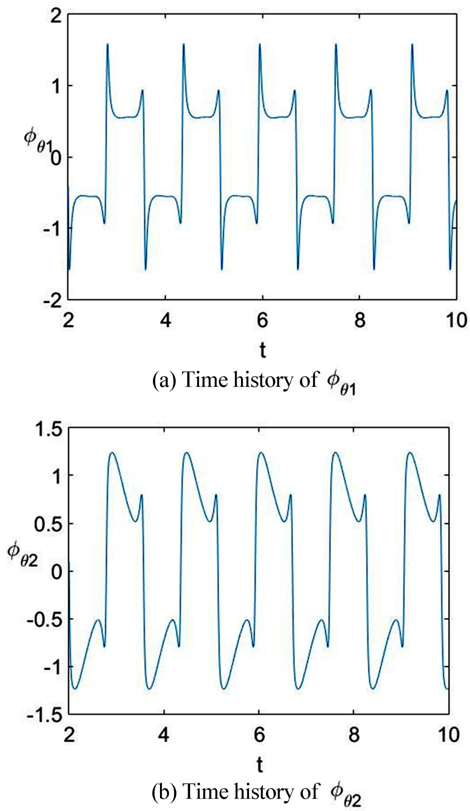


(b) Time history of w_2

图 5 叶片振动波形图

Fig.5 The vibration waveform of the blade

选定参数 $M_1=1.3$,画出系统的波形图进行研究.(a)(b)分别是(t,φ_{θ_1})、(t,φ_{θ_2})平面上的波形图.



(a) Time history of $\phi_{\theta 1}$

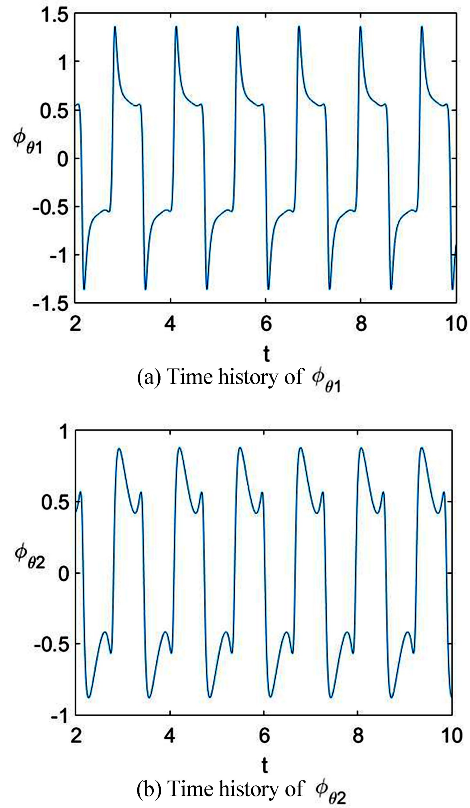
(b) Time history of $\phi_{\theta 2}$

图 6　叶片振动波形图($M_1=1.3$)

Fig.6　The vibration waveform of the blade($M_1=1.3$)

选定参数 $M_1=1.67$,画出系统的波形图进行研究.(a)(b)分别是(t,φ_{θ_1})、(t,φ_{θ_2})平面上的波形图.

旋转悬臂圆锥壳上的横向扭矩变化时,φ_{θ_1}和φ_{θ_2}的幅值有变化.当增大参数 M_1 时,系统的振动幅值也同时增大.



(a) Time history of $\phi_{\theta 1}$

(b) Time history of $\phi_{\theta 2}$

图 7　叶片振动波形图($M_1=1.67$)

Fig.7　The vibration waveform of the blade($M_1=1.67$)

4　结论

本文提供了一个分析模型,用来研究变转速条件下航空发动机叶片的非线性动力学响应.基于一阶剪切变形理论,利用 Hamilton 原理建立了悬臂圆锥壳的非线性动力学方程.利用 Galerkin 方法,对偏微分动力学方程进行了离散,转化为常微分方程组.利用数值方法,研究了转速和外界激励对叶片振动的影响.从理论分析和数值结果可以得到下列结论:

(1)转速对航空发动机叶片的振动幅值有影响,伴随旋转速度的增加,振动幅值减小;

(2)横向外力对航空发动机叶片的振动有影响,力的增大改变振幅;

(3)横向扭矩对航空发动机叶片的振动幅值有影响,伴随扭矩的增加,振动幅值增大.

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NONLINEAR VIBRATION ANALYSIS OF THE AERO-ENGINE BLADE *

Jie Xiaobo Zhang Wei†

(*College of Mechanical Engineering*, *Beijing University of Technology*, *Beijing* 100124, *China*)

Abstract The nonlinear vibration responses of the aero-engine blade were investigated. Considering the effects of the presetting and pre-twisted angles, the blade was modeled as a rotating conical shell . Using Hamilton's principle, the governing partial differential equations of motion under different excitations were established. Both Galerkin's approach and numerical method were applied to analyze the nonlinear dynamics of the blade. Numerical simulations were performed to investigate the responses of the blade under different rotating speeds and excitations. The results show that both the rotating speed and excitation have effects on the nonlinear dynamics of the blade.

Key words rotating blade, nonlinear dynamics, conical shell, Hamilton's principle, Galerkin's method

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† Corresponding author E-mail:iron_man1945@163.com