第 6 卷第 3 期 2008 年 9 月
1672-6553/2008/06(3)/243-6
动 力 学 与 控 制 学 报
JOURNAL OF DYNAMICS AND CONTROL
Vol. 6 No. 3
Sep. 2008

变流速输液管的周期和混沌振动*

席红敏　张伟　姚明辉
（北京工业大学机电学院，北京　100022）

摘要　研究了参数激励和外激励联合作用下输流管道的非线性振动问题. 只考虑管道变形的几何非线性因素，利用 Hamilton 原理得到单侧受简谐均布载荷作用下输液管的非线性动力学方程，对系统运动偏微分方程综合运用多尺度法和 Galerkin 离散方法，得到了主参数共振-1/2 亚谐共振和 1∶2 内共振情况下的平均方程. 数值模拟结果表明参数激励和外激励联合作用下的悬臂输液管呈现周期运动、多倍周期运动和混沌运动的变化规律.

关键词　输液管，　变流速，　非线性振动，　混沌运动

引言

输液管道在众多工业领域内具有广泛的应用，关于液体流动引起输液管振动的研究有着广阔的工程应用背景，它的研究成果可以直接应用于航空航天工程、核工业、海底输油管道工程、动力工程、生物工程等领域. 众所周知，管道系统工作过程中不可避免的会由于扰动而产生非定常流动，从而引起管道振动，而较长输液管道由于柔性的影响，易发生较大振幅的振动，需要用非线性动力学的理论来进行研究，近年来，输液管系统非线性振动问题已引起越来越多的关注，逐渐成为非线性动力学领域研究的热点[1-3].

1994 年，Semeler 等人[4]研究了小阻尼对悬臂输液管的动力稳定性的影响，并得出不同坐标的相位角是影响系统稳定性的重要原因. 1996 年，Semeler 等人[5]研究了流速具有谐波增量项的竖直悬臂输液管参数激励下的动力学响应. 1997 年，金基铎等人[6,7]对具有弹性约束的悬臂输液管的颤振和混沌运动进行了分析，发现当流速较大、支承刚度很小时，管道可能发生颤振而动态失稳，混沌运动可能发生，混沌区域随着刚度值增大而逐渐变窄. 1998 年，Paidoussis 和 Semler[8]对自由端带质量块的竖直悬臂输液管的非线性动力学特性分别进行了理论分析和实验研究. 1998 年，Paidoussis[9]将其对各类输液管道振动的非线性动力学问题的研究成果整理成文，出版了《Fluid-Structure Interactions (Slender Structures and Axial Flow), VOL1》. 2000 年，徐鉴和黄玉盈[10,11]研究了自由端带喷嘴的悬臂梁的动力特性，得到分别与线性刚度、喷嘴参数和流速相关的三个临界值. 2002 年，张立翔和黄文虎[12]将弱约束输液管道非定常液流固耦合运动按波-流-振动系统建模成由 4 个非线性微分方程组成的分析模型，研究系统在多种耦合状态下具有的运动稳定特性. 2003 年，金基铎和邹光胜[13]进一步研究了带有弹性约束的变流速悬臂输液管局部分叉，并将分析结果与数值计算结果进行了比较. 2004 年，Sarkar 和 Paidoussis[14]研究比较了自由端有质量块的竖直悬臂输液管与悬臂梁的固有模态. 2004 年，王琳和倪樵[15]把微分求积法应用到输液管道的非线性动力学分析. 2006 年，王琳等人[16]研究了考虑内流压力时输流曲管的混沌运动. 2007 年，包日东和闻邦椿[17]分析了两端扭转弹簧约束下简支输流管的非线性动力学.

研究了参数激励和外激励联合作用下的输流管道的非线性振动问题. 仅考虑管道变形的几何非线性因素，不考虑管道材料的本构非线性因素，建立了单侧受简谐均布载荷作用的脉动流输液管竖直悬臂梁的动力学控制方程，综合运用多尺度法和 Galerkin 离散方法，研究了该系统的周期和混沌振动.

1　悬臂输液管的运动控制方程

根据文献[1]，建立了脉动流作用下，单侧受简

2008-04-21 收到第 1 稿，2008-05-15 收到修改稿.
* 国家自然科学基金资助项目(10732020，10425209)

谐均布载荷作用的竖直悬臂输液管道的非线性运动方程,具体模型如图 1 所示.

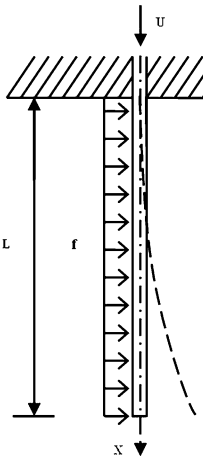

图 1　竖直悬臂 Euler-Bernoulli 型输液管模型

Fig. 1　Fluid-converying pipes model of hanging Euler-Bernoulli cantilever

在该模型中,我们作了如下假设:管道内为不可压缩流体;管道平面运动大变形;忽略管道的旋转和剪切;且对于悬臂梁模型,假设轴向无伸长.

并引入沿中线 s 的曲线坐标系(s,t),如图 2 所示.

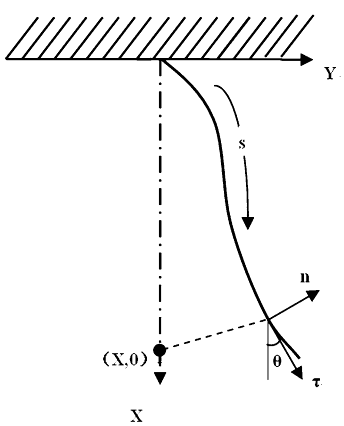

图 2　曲线坐标系下的输液管模型

Fig. 2　Fluid-conveying pipes model in curvilinear coordinate system

基于 Hamilton 原理,可以得到系统的运动控制方程如下

$$(m+M)\ddot{y}+2MU\dot{y}'(1+y'^2)+(m+M)gy'(1+\frac{1}{2}y'^2)+y''[MU^2(1+y'^2)+(M\dot{U}-(m+M)g)(L\text{-}s)(1+\frac{1}{2}y'^2)]+EI(y''''+4y'y''y'''+y''^3+y''''y'^2)\text{-}y''[\int_s^L\int_0^s(m+M)(\dot{y}'^2+y'\ddot{y}')\mathrm{d}s\mathrm{d}s+\int_s^L(\frac{1}{2}M\dot{U}y'^2+2MUy'\dot{y}'+MU^2y'y'')\mathrm{d}s]+y'\int_0^s(m+M)(\dot{y}'^2+y'\ddot{y}')\mathrm{d}s+f=0 \quad (1)$$

其中,m 为单位长度的管道质量,M 为单位长度流体质量,U 为管内流体的流速,E 为管道材料的弹性模量,I 为管道截面惯性矩,y 为管道横向位移,"′"表示对坐标的导数,变量上方的"·"代表对时间的导数.

令

$$\xi=\frac{s}{L},\eta=\frac{y}{L},\tau=\left(\frac{EI}{m+M}\right)^{1/2}\frac{t}{L^2},$$

$$U^*=\left(\frac{M}{EI}\right)^{1/2}UL,\gamma=\frac{m+M}{EI}L^3g,$$

$$\beta=\frac{m}{m+M},f^*=\frac{L^3}{EI}f. \quad (2)$$

将式(2)代入方程(1)中,为了书写方便,去掉 * 号,得到无量纲方程如下

$$\eta''''+\ddot{\eta}+2U\sqrt{\beta}\dot{\eta}'(1+\eta'^2)+\eta''[U^2(1+\eta'^2)+(\dot{U}\sqrt{\beta}-\gamma)(1-\xi)(1+\frac{3}{2}\eta'^2)]+\gamma\eta'(1+\frac{1}{2}\eta'^2)+\eta''''\eta'^2+4\eta'\eta''\eta'''+\eta''^3\text{-}\eta''[\int_\xi^1\int_0^\xi(\dot{\eta}'^2+\eta'\ddot{\eta}')\mathrm{d}\xi\mathrm{d}\xi+\int_\xi^1(\frac{1}{2}\dot{U}\sqrt{\beta}\eta'^2+2U\sqrt{\beta}\eta'\dot{\eta}'+U^2\eta'\eta'')\mathrm{d}\xi]+\eta''\int_0^\xi(\dot{\eta}'^2+\eta'\ddot{\eta}')\mathrm{d}\xi+f=0 \quad (3)$$

2　摄动分析和 Galerkin 离散

引入质量、陀螺和线性刚度算子

$$M=I,G=2U_0\sqrt{\beta}\frac{\partial}{\partial x},$$

$$K=[U_0^2-\gamma(1-\xi)]\frac{\partial}{\partial x^2}+\gamma\frac{\partial}{\partial x}+\frac{\partial}{\partial x^4} \quad (4)$$

在该系统中,我们考虑管道内为变流速流体 $U=U_0+U_1\cos\Omega_1 t$,外载荷 f 是简谐载荷 $f=f_0+f_1\cos\Omega_2 t$. 为了进行摄动分析,引入如下变换

$$U\rightarrow U_0+\varepsilon U_1\cos\Omega_1 t,f\rightarrow\varepsilon f,N(\eta)\rightarrow\varepsilon N(\eta) \quad (5)$$

忽略因引入 $U\rightarrow U_0+\varepsilon U_1\cos\Omega_1 t$ 后所出现的四次及以上非线性项,并去掉无量纲的 * 号,于是方程(3)可以写作

$$M\ddot{\eta}+G\dot{\eta}+K\eta=\varepsilon F(\eta)-\varepsilon N(\eta)-2\varepsilon U_1\sqrt{\beta}\dot{\eta}'\cos\Omega_1 t-2\varepsilon\eta''U_0U_1\cos\Omega_1 t-\varepsilon\eta''\Omega_1U_1\sqrt{\beta}(1-\xi)\sin\Omega_1 t \quad (6)$$

其中

$$F(\eta) = -f_0 - f_1\cos\Omega_2 t \tag{7}$$

$$N(\eta) = 2U_0\sqrt{\beta}\dot{\eta}'\eta'^2 + \eta''\eta'^2[U_0^2 - \frac{3}{2}\gamma(1-\xi)] + \frac{1}{2}\gamma\eta'^3 + \eta''''\eta'^2 + 4\eta'\eta''\eta''' + \eta''^3 - \eta''\int_\xi^1\int_0^\xi(\dot{\eta}'^2 + \eta'\ddot{\eta}')\mathrm{d}\xi\mathrm{d}\xi - \eta''\int_\xi^1(2U_0\sqrt{\beta}\eta'\dot{\eta}' + U_0^2\eta'\eta'')\mathrm{d}\xi + \eta'\int_0^\xi(\dot{\eta}'^2 + \eta'\ddot{\eta}')\mathrm{d}\xi \tag{8}$$

对方程(6)运用多尺度方法[18]进行摄动分析,设一致渐近解的形式为

$$\eta(t,\varepsilon) = \eta_0(T_0,T_1) + \varepsilon\eta_1(T_0,T_1) \tag{9}$$

其中 $T_0 = t, T_1 = \varepsilon t$.

将(9)式代入方程(6)中,比较方程两端 ε 次幂的系数,得到

$$\varepsilon^0: MD_0^2\eta_0 + G(D_0\eta_0) + K\eta_0 = 0 \tag{10}$$

$$\varepsilon^1: MD_0^2\eta_1 + GD_0\eta_1 + K\eta_1 = -f_0 - f_1\cos\Omega_2 t - 2U_0\sqrt{\beta}\frac{\partial^2\eta_0}{\partial T_0\partial X}\left(\frac{\partial\eta_0}{\partial X}\right)^2 - \frac{\partial^2\eta_0}{\partial X^2}\left(\frac{\partial\eta_0}{\partial X}\right)^2 \times \left(U_0^2 - \frac{3}{2}\gamma(1-\xi)\right) - \frac{1}{2}\gamma\left(\frac{\partial\eta_0}{\partial X}\right)^3 - \left[\frac{\partial^4\eta_0}{\partial X^4}\left(\frac{\partial\eta_0}{\partial X}\right)^2 + 4\frac{\partial\eta_0}{\partial X}\frac{\partial^2\eta_0}{\partial X^2}\frac{\partial^3\eta_0}{\partial X^3} + \left(\frac{\partial^2\eta_0}{\partial X^2}\right)^3\right] + \frac{\partial^2\eta_0}{\partial X^2}\left[\int_\xi^1\int_0^\xi\left(\left(\frac{\partial^2\eta_0}{\partial T_0\partial X}\right)^2 + \frac{\partial\eta_0}{\partial X}\frac{\partial^3\eta_0}{\partial T_0^2\partial X}\right)\mathrm{d}\xi\mathrm{d}\xi + \int_0^\xi\left(2U_0\sqrt{\beta}\frac{\partial\eta_0}{\partial X}\frac{\partial^2\eta_0}{\partial T_0\partial X} + U_0^2\frac{\partial\eta_0}{\partial X}\frac{\partial^2\eta_0}{\partial X^2}\right)\mathrm{d}\xi\right] - \frac{\partial\eta_0}{\partial X}\int_0^\xi\left(\frac{\partial^2\eta_0}{\partial T_0\partial X} + \frac{\partial\eta_0}{\partial X}\frac{\partial^3\eta_0}{\partial T_0^2\partial X}\right)\mathrm{d}\xi - 2U_1\sqrt{\beta}\frac{\partial^2\eta_0}{\partial T_0\partial X}\cos\Omega_1 t - \frac{\partial^2\eta_0}{\partial X^2}[2U_0U_1\cos\Omega_1 t - \Omega_1 U_1\sqrt{\beta}(1-\xi)\sin\Omega_1 t] - 2MD_0D_1\eta_0 - GD_1\eta_0 \tag{11}$$

引进模态函数,对系统进行二阶 Galerkin 离散,方程(10)的通解可用复数形式表示为

$$\eta(T_0,T_1,x) = \varphi_1(x)A_1(T_1)e^{i\omega_1T_0} + \varphi_2(x)A_2(T_1)e^{i\omega_2T_0} + cc \tag{12}$$

这里 cc 是前一项的复共轭.

式(12)中 $\varphi(j=1,2)$ 为悬臂梁的模态函数,用它近似代替相同边界条件下输流管道的模态函数,则有

$$\varphi_j(\xi) = \cosh(\beta_j x) - \cos(\beta_j x) - \lambda_j[\sinh(\beta_j x) - \sin(\beta_j x)] \tag{13}$$

其中

$$\lambda_j = \frac{\sinh(\beta_j) - \sin(\beta_j)}{\cosh(\beta_j) - \cos(\beta_j)}, \beta_1 = 1.875, \beta_2 = 4.694$$

考虑 1:2 内共振和主参数共振的情形

$$\omega_1 = \frac{1}{2}\Omega_1 + \varepsilon\sigma_1, \omega_2 = \Omega_1 + \varepsilon\sigma_2, \Omega_1 = \Omega_2 \tag{14}$$

这里 σ_1 和 σ_2 是两个谐调参数.

将(13)和(14)式代入方程(11),得到

$$MD_0^2\eta_1 + GD_0\eta_1 + K\eta_1 = \Gamma_1 e^{iT_0(\frac{1}{2}\Omega_1+\varepsilon\sigma_1)} + \Gamma_2 e^{iT_0(\Omega_1+\varepsilon\sigma_2)} - f_1 e^{iT_0\Omega_1} + [\frac{1}{2}iU_1\sqrt{\beta}A_{12}\Omega_1(\varphi_1' - \varphi_1'' - \xi\varphi_1'') - U_0U_1A_{12}\varphi_1'']e^{iT_0(\frac{1}{2}\Omega_1-\varepsilon\sigma_1)} + cc + NST \tag{15}$$

这里 NST 代表不产生长期项的所有项,并且有

$$\Gamma_1 = i\Omega A_{11}^2A_{12}U_0\sqrt{\beta}m_1 + 2i\Omega A_{11}A_{21}A_{22}U_0\sqrt{\beta}m_2 + A_{11}^2A_{12}(3U_0^2m_3 - \gamma m_4 - 3m_5 + \frac{3}{4}\Omega^2m_6) + A_{11}A_{21}A_{22}(U_0^2m_7 - \gamma m_8 + m_9 + \Omega^2m_{10}) - i\Omega_1\dot{A}_{11}M\varphi_1 - \dot{A}_{11}G\varphi_1 \tag{16a}$$

$$\Gamma_2 = i\Omega A_{21}^2A_{22}U_0\sqrt{\beta}n_1 + 4i\Omega A_{11}A_{12}A_{21}U_0\sqrt{\beta}n_2 + A_{21}^2A_{22}(3U_0^2n_3 - \gamma n_4 - 3n_5 + 3\Omega^2n_6) + A_{11}A_{12}A_{21}(U_0^2n_7 - \gamma n_8 + n_9 + \Omega^2n_{10}) - 2i\Omega_1\dot{A}_{21}M\varphi_2 - \dot{A}_{21}G\varphi_2 \tag{16b}$$

当非齐次方程(15)满足可解性条件时,方程(15)有解. 因此,可解性条件要求方程(15)的右端与其对应的齐次伴随方程的解正交,所以有

$$\int_0^1(\Gamma_1(T_1,x) + (\frac{1}{2}iU_1\sqrt{\beta}A_{12}\Omega_1(\varphi_1' - \varphi_1'' - \xi\varphi_1'') - U_0U_1A_{12}\varphi_1'')e^{-2i\sigma_1T_1})\bar{\varphi}_1\mathrm{d}x = 0 \tag{17a}$$

$$\int_0^1(\Gamma_1(T_1,x) - f_1e^{-i\sigma_2T_1})\bar{\varphi}_2\mathrm{d}x = 0 \tag{17b}$$

令

$$A_1(T_1) = [x_1(T_1) - ix_2(T_1)]e^{-i(\sigma_1T_1 - 2m\pi)},$$
$$A_2(T_1) = [x_3(T_1) - ix_4(T_1)]e^{-i(\sigma_2T_1 - 2n\pi)} \tag{18}$$

将(18)式代入方程(17),经过化简,并分离实部和虚部,可以得到系统的平均方程如下,

$$\dot{x}_1 = U_1\sqrt{\beta}\Omega p_{11}x_1 - \mu x_1 - U_0U_1p_{12}x_2 - \sigma_1x_2 + U_0\sqrt{\beta}p_1x_1(x_1^2 + x_2^2) + 2U_0\sqrt{\beta}p_2x_1(x_3^2 + x_4^2) - (3U_0^2p_3 - \gamma p_4 - 3p_5 + \frac{3}{4}\Omega^2p_6)x_2(x_1^2 + x_2^2) -$$

$$(U_0^2 p_7 - \gamma p_8 + p_9 + \Omega^2 p_{10}) x_2 (x_3^2 + x_4^2) \quad (19a)$$

$$\begin{aligned}\dot{x}_2 = & -U_1\sqrt{\beta}\Omega p_{11} x_2 - \mu x_2 - U_0 U_1 p_{12} x_1 + \sigma_1 x_1 + \\ & U_0\sqrt{\beta} p_1 x_2 (x_1^2 + x_2^2) + 2U_0\sqrt{\beta} p_2 x_2 (x_3^2 + x_4^2) + \\ & (3U_0^2 p_3 - \gamma p_4 - 3p_5 + \frac{3}{4}\Omega^2 p_6) x_1 (x_1^2 + x_2^2) + \\ & (U_0^2 p_7 - \gamma p_8 + p_9 + \Omega^2 p_{10}) x_1 (x_3^2 + x_4^2) \quad (19b)\end{aligned}$$

$$\begin{aligned}\dot{x}_3 = & -\mu x_3 - \sigma_2 x_4 + U_0\sqrt{\beta}\Omega q_1 x_3 (x_3^2 + x_4^2) + \\ & 4U_0\sqrt{\beta}\Omega q_2 x_3 (x_1^2 + x_2^2) - (3U_0^2 q_3 - \gamma q_4 - q_5 + \\ & 3\Omega^2 q_6) x_4 (x_3^2 + x_4^2) - (U_0^2 q_7 - \gamma q_8 + q_9 + \\ & \Omega^2 q_{10}) x_4 (x_1^2 + x_2^2) \quad (19c)\end{aligned}$$

$$\begin{aligned}\dot{x}_4 = & -f_1 - \mu x_4 + \sigma_2 x_3 - U_0\sqrt{\beta}\Omega q_1 x_4 (x_3^2 + x_4^2) - \\ & 4U_0\sqrt{\beta}\Omega q_2 x_4 (x_1^2 + x_2^2) + (3U_0^2 q_3 - \gamma q_4 - \\ & q_5 + 3\Omega^2 q_6) x_3 (x_3^2 + x_4^2) + (U_0^2 q_7 - \gamma q_8 + \\ & q_9 + \Omega^2 q_{10}) x_3 (x_1^2 + x_2^2) \quad (19d)\end{aligned}$$

3　数值模拟

利用四阶 Runge-Kutta 法对所得到的平均方程(19)进行数值模拟. 通过相图和波形图可以反映系统的非线性动力学行为. 系统所选取的参数和初值分别为: $L = 30\text{m}$, $\rho_1 = 7850\text{kg/m}^3$, $\rho_2 = 1000\text{kg/m}^3$, $E = 200\times10^9$, $d = 15\times10^{-3}\text{m}$, $\delta = 2.5\times10^{-3}\text{m}$, $f_1 = 1\text{N/m}$, $\Omega = 2\pi\times50$, $\sigma_1 = 12.1$, $\sigma_2 = 9.8$, $\mu = 0.02$, $x_{10} = 0.0233$, $x_{20} = 0.0753$, $x_{30} = 0.0325$, $x_{40} = -0.0663$, $U_0 : U_1$ 为 5∶1. 只改变管内流速, 当管内流速为 17m/s 时, 系统的响应为单倍周期运动, 如图 3 所示, 其中图 3(a)为(x_1, x_2)平面相图, 图 3(b)为(x_3, x_4)平面相图, 图 3(c)为三维相图, 图 3(d)和图 3(e)为波形图, 图 3(f)为频谱图.

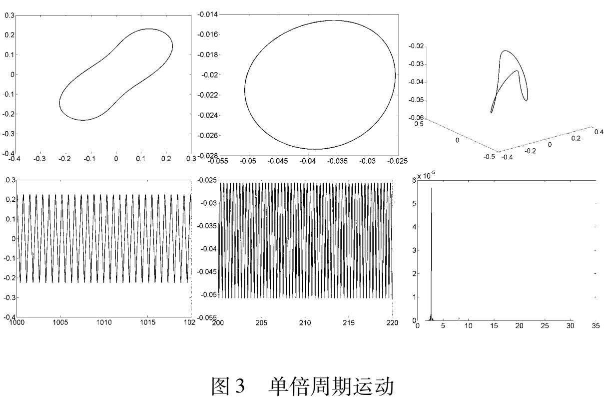

图 3　单倍周期运动

Fig. 3　Period-1 solution of the fluid-conveying pipe

图 4 表明当管内流速为 27.5m/s 时, 系统发生多倍周期运动. 当流速增大到 30m/s 时, 系统由多倍周期运动演化为混沌运动, 如图 5 所示.

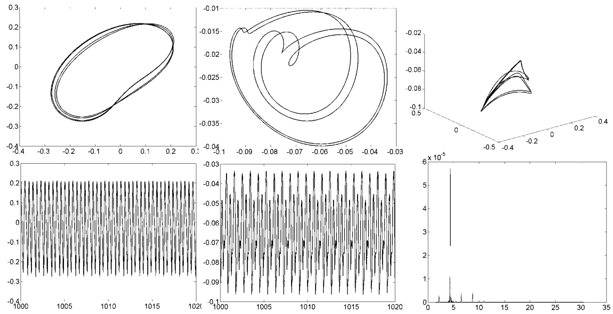

图 4　多倍周期运动

Fig. 4　Periodic-n motion of the fluid-conveying pipe

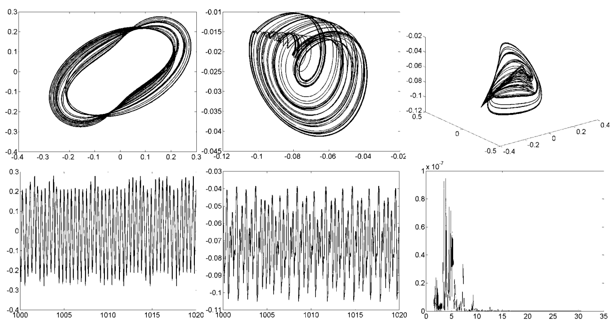

图 5　混沌运动

Fig. 5　Chaotic motion of the fluid-conveying pipe

当流速增大至 31.5m/s 时, 系统又回到多倍周期运动, 如图 6 所示.

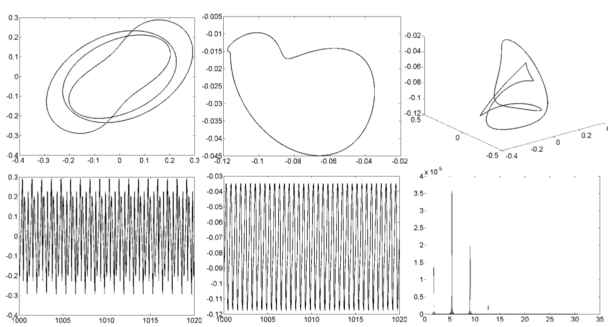

图 6　多倍周期运动

Fig. 6　Periodic-n motion of the fluid-conveying pipe

图 7 表明流速增加到 34m/s 时, 系统又由多倍周期运动演化为混沌运动.

从前面的数值结果和图 8 可以看出, 在图 3 至图 7 中, 所取流速值情况下系统的响应与图 8 所示的分叉图可以很好的吻合. 根据上述数值模拟结果, 并结合图 8 所示以流速 U_0 为控制参数的分叉图(作图时选取 $\dot{x}_1 = 0$ 时 x_1 的对应值), 可以发现, 随着管道流速的增加, 系统通过倍周期分叉进入混沌运动区域, 之后系统出现一段周期窗口, 然后又

进入混沌运动区域.

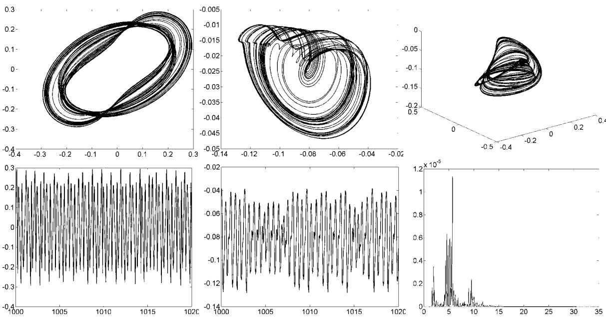

图 7　混沌运动

Fig. 7　Chaotic motion of the fluid-conveying pipe

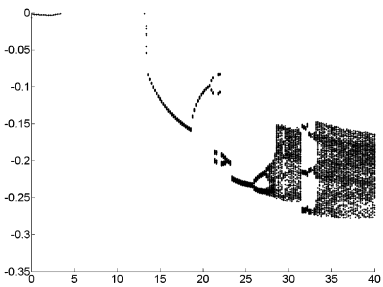

图 8　分叉图

Fig. 8　Bifurcation diagram of the fluid-conveying pipe

4　结论

本文在考虑管道变形的几何非线性情况下,建立了竖直悬臂输液管道在脉动流和外激励共同作用下的非线性动力学控制方程,综合应用多尺度法和 Galerkin 离散方法对系统在 1:2 内共振和主参数共振情况下的动力学响应进行了分析. 数值模拟发现参数激励和外激励共同作用下的悬臂输液管道存在周期和混沌运动. 当系统其它参数固定不变,只改变管内流体速度及管内流体脉动项时,系统呈现出由周期运动→多倍周期运动→混沌运动→多倍周期运动→周期运动的变化规律. 可以通过改变流速及脉动项来控制输液管道的非线性动力学响应,使得系统从不稳定运动状态进入稳定运动状态. 该研究为实际工程提供了理论依据.

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PERIODIC AND CHAOTIC OSCILLATIONS OF THE FLUID CONVEYING PIPES WITH PULSE FLUID *

Xi Hongmin Zhang Wei Yao Minghui

(*College of Mechanical Engineering*, *University of Technology*, *Beijing* 100022, *China*)

Abstract The nonlinear oscillations of fluid conveying pipes under parametric and external excitations were researched. The nonlinear governing equation of motion for the cantilevered pipes conveying pulse fluid with one-sided harmonic external force was obtained by using the Hamilton principle, in which the nonlinear geometric deformation of the pipe was only considered. The method of multiple scales and Galerkin method were employed to transform the partial differential governing equation of motion to the average equations in the case of the principal parametric resonance-1/2 subharmonic resonance and 1:2 internal resonance. The results of numerical simulation indicate that there exist the periodic motion and chaotic motion in the conveying fluid pipes under parametric and external excitation.

Key words fluid conveying pipe, pulse fluid, nonlinear oscillations, chaotic motion

Received 21 April 2008, revised 15 May 2008.

* The project supported by the Natural Science Foundation of China (10732020, 10425209)