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非线性桩基的动力响应分析*

花羽超[1]　朱媛媛[2]　程昌钧[1]

（1. 上海大学力学系,上海市应用数学和力学研究所,上海　200072）（2. 上海师范大学计算机科学与技术系,上海　200234）

摘要　假定桩基材料服从一种 3 次非线性本构关系,同时桩被置于弹性基础上,基于 Timoshenko 梁的修正理论和广义 Hamilton 变分原理,建立了非线性弹性桩基动力学行为分析的数学模型, 该模型包括了 3 个位移和 2 个转动. 作为该数学模型的应用,在空间上采用中心差分格式,在时间上采用向后差分格式,计算和分析了非线性弹性桩基受轴力作用时的轴向运动和非线性桩基的平面耦合运动,得到了数值解,考察了非线性弹性性质对结构动力学响应的影响.

关键词　非线性弹性材料,　Hamilton 变分原理,　非线性桩基耦合运动,　动力学响应,　非线性弹性参数的影响

引 言

桩基是一种重要的工程结构,已广泛地应用于土木和环境等工程中,近年来吸引了许多研究者的注意,特别是桩基的非线性动力学行为分析. 在大多数情况下,由于高应变下的土的非线性、桩—土的剥离和摩擦,桩 - 土的相互作用以及载荷的传递过程等因素,桩基的力学行为分析一般是非常复杂的,采用连续介质力学模型进行研究也是比较困难的,许多理论与实验研究是基于梁 - 柱的理论来研究桩基的动力学行为. 王奎华[1-2]曾得到粘弹性桩基在瞬态或稳态激励力下的时域纵向振动响应解析解. 贾启芬等[3]研究了桩基结构非线性复杂动力学行为,得到双曲型模型粘弹性土中,粘弹性桩在非线性参数激励下的动态响应曲线的拓扑结构分叉图,并讨论了系统参数对结构稳定性的影响. 王宏志等[4]得到了分层土中弹性桩纵向振动的半解析解. 禹金云[5]分析了单桩在轴向扰力作用下的混沌运动. 任九生和程昌钧[6]分析了在地基波动影响下非线性粘弹性桩的混沌运动. Novak [7]利用小模型桩进行了桩基动力特性试验研究. Reese [8]、O' Neill [9]通过由经验或实验得到的土对桩的特殊曲线来描述土的非线性作用,对桩 - 土的非线性行为进行了研究. Nabil [10]和 Rollins [11]等用试验方法研究了受横向荷载作用的砂土和粘土中的桩的特性. 但是,关于非线性桩基动力学响应分析的工作报道不多.

本文的目的是研究非线性弹性材料桩基的动力学响应. 首先,基于梁的 Timoshenko 假设和 Hamilton 原理建立了用 5 个广义位移表示的非线性弹性桩基动力学响应分析的数学模型,其中,假设变形是小的,桩基的材料由一种 3 次非线性弹性的本构关系来描述,同时桩基位于线性弹性的基础上,采用 Winkler 模型来模拟基础对桩基的作用. 由于材料的非线性,因此,控制微分方程和力的边界条件都是非线性的. 作为数学模型的应用,利用有限差分法求解了非线性桩基的轴向运动和平面耦合运动,给出了问题的数值解,得到了位移随时间和空间的变化,同时考察了非线性弹性性质对桩基动力学响应的影响.

1　基本假设

考察图 1 所示置于弹性基础上的承受任意横向载荷作用的结构的动力学问题. 设轴为截面的中性轴,、为截面的惯性主轴. 假设结构是等截面的,面积为,长为,密度为,作用于结构上的载荷的 - 和 - 分量分别为和,同时,在端部受轴向拉力的作用. 根据 Timoshenko 梁理论,有如下的位移场

$$u_1 = u(x,t) + y\varphi(x,t) + z\psi(x,t),$$
$$v_1 = v(x,t),\quad w_1 = w(x,t) \tag{1}$$

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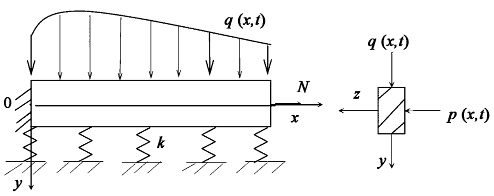

图 1　梁模型

Fig. 1　Model of beam

其中,u,v,w 是中性轴的位移,φ,ψ 是截面的转动 y,z 是到中性轴的坐标.

几何关系:当变形较小时,非零的应变分量为

$$\varepsilon_x=\frac{\partial u}{\partial x}+y\frac{\partial\varphi}{\partial x}+z\frac{\partial\psi}{\partial x},\gamma_{xy}=\frac{\partial v}{\partial x}+\varphi,\gamma_{xz}=\frac{\partial w}{\partial x}+\psi \tag{2}$$

本构关系:假设桩的材料是一种非线性弹性材料,并有如下本构关系:

$$\sigma=E_1\varepsilon+E_2\varepsilon^2+E_3\varepsilon^3,\tau_{xz}=G\gamma_{xz},\tau_{xy}=G\gamma_{xy} \tag{3}$$

式中,E_1 是材料的弹性模量,E_2 和 E_3 是考虑材料非线性影响的广义弹性模量. 这种本构关系可以描述某种混凝土材料的力学特性[12].

2　Hamilton 变分原理

利用 Hamilton 变分原理来推导非线性弹性桩基的位移 u,v,w,φ,ψ 满足的运动微分方程和边界条件.

Hamilton 变分原理:在满足几何方程、位移边界条件,并在初始时刻和终止时刻具有指定运动的一切可能位移中,真实的位移 u,v,w,φ,ψ 使泛函

$$\Pi=\int_0^T H\mathrm{d}t=\int_0^T(U_1+U_2-W-T)\mathrm{d}t \tag{4}$$

取驻值,其中:T 为结构的动能,U_1 和 U_2 分别为由正应变,剪应变引起的应变能,W 表示横向外载荷、轴向拉力和基础反力所作的功. 它们分别定义为

$$T=\int_0^l\frac{1}{2}\rho\{A((\frac{\partial u}{\partial t})^2+(\frac{\partial v}{\partial t})^2+(\frac{\partial w}{\partial t})^2)+I_z(\frac{\partial\varphi}{\partial t})^2+I_y(\frac{\partial\psi}{\partial t})^2\}\mathrm{d}x \tag{5a}$$

$$\begin{aligned}U_1=\int_0^l\{&\frac{A}{2}E_1(\frac{\partial u}{\partial x})^2+\frac{3}{16}AE_2(\frac{\partial u}{\partial x})^3+\frac{A}{4}E_3(\frac{\partial u}{\partial x})^4+\\&\frac{1}{2}I_zE_1(\frac{\partial\varphi}{\partial x})^2+I_zE_2(\frac{\partial\varphi}{\partial x})^2\frac{\partial u}{\partial x}+\frac{3}{2}I_zE_3(\frac{\partial u}{\partial x})^2(\frac{\partial\varphi}{\partial x})^2+\\&\frac{1}{4}\bar{I}_zE_3(\frac{\partial\varphi}{\partial x})^4+\frac{1}{2}I_yE_1(\frac{\partial\psi}{\partial x})^2+I_yE_2(\frac{\partial\psi}{\partial x})^2\frac{\partial u}{\partial x}+\\&\frac{3}{2}I_yE_3(\frac{\partial u}{\partial x})^2(\frac{\partial\psi}{\partial x})^2+\frac{1}{4}\bar{I}_yE_3(\frac{\partial\psi}{\partial x})^4+\\&\frac{3}{2}I_{xy}E_3(\frac{\partial\varphi}{\partial x})^2(\frac{\partial\psi}{\partial x})^2\}\mathrm{d}x\end{aligned} \tag{5b}$$

$$\begin{aligned}U_2=\int_0^l[&\frac{A}{2}Gk_1\varphi^2+\frac{A}{2}Gk_2\psi^2+AGk_1\varphi\frac{\partial v}{\partial x}+\\&\frac{A}{2}Gk_1(\frac{\partial v}{\partial x})^2+AGk_1\psi\frac{\partial w}{\partial x}+\frac{A}{2}Gk_2(\frac{\partial w}{\partial x})^2]\mathrm{d}x\end{aligned} \tag{5c}$$

$$\begin{aligned}W=&\int_0^l(qv+pw)\mathrm{d}x+Nu(l)-\frac{N}{2}\int_0^l[(\frac{\partial v}{\partial x})^2+\\&(\frac{\partial w}{\partial x})^2]\mathrm{d}x-\frac{k}{2}\int_0^l[v^2+w^2]\mathrm{d}x\end{aligned} \tag{5d}$$

式中,$I_y,I_z,I_{yz},\bar{I}_y$ 和 $\bar{I}_z$ 分别为横截面对 y 轴和 z 轴的惯性矩和高阶惯性矩,即 $I_y=\iint_A z^2\mathrm{d}A,I_z=\iint_A y^2\mathrm{d}A,I_{yz}=\iint_A z^2y^2\mathrm{d}A,\bar{I}_y=\iint_A z^4\mathrm{d}A,\bar{I}_z=\iint_A y^4\mathrm{d}A$;$\kappa_1$ 和 κ_2 是横截面在 y 和 z 方向的剪力修正系数,它们表示对“横截面上剪切应变为常数”的假定的修正,k 为基础的刚度系数.

对各量进行变分,将其结果代入 $\delta\Pi=0$ 中,注意到在初始时刻和终止时刻结构具有指定运动,因此,在 $t=0$ 和 $t=T$ 时 $\delta u,\delta v,\delta w,\delta\varphi,\delta\psi$ 为零. 同时注意到在区域$[0,l]$中 $\delta u,\delta v,\delta w,\delta\varphi,\delta\psi$ 的任意性,在力端 $\delta u,\delta v,\delta w,\delta\varphi,\delta\psi$ 的任意性,则可推得广义位移 u,v,w,φ,ψ 满足的运动微分方程和端部力的边界条件.

运动微分方程:

$$\begin{aligned}&AE_1\frac{\partial^2u}{\partial x^2}+2AE_2\frac{\partial u\partial^2u}{\partial x\partial x^2}+3AE_3(\frac{\partial u}{\partial x})^2\frac{\partial^2u}{\partial x^2}+2I_zE_2\frac{\partial\varphi}{\partial x}\frac{\partial^2\varphi}{\partial x^2}+\\&2I_yE_2\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial x^2}+6I_zE_3\frac{\partial\varphi}{\partial x}\frac{\partial^2\varphi}{\partial x^2}\frac{\partial u}{\partial x}+3I_zE_3\frac{\partial^2u}{\partial x^2}(\frac{\partial\varphi}{\partial x})^2+\\&6I_yE_3\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial x^2}\frac{\partial u}{\partial x}+3I_zE_3\frac{\partial^2u}{\partial x^2}(\frac{\partial\psi}{\partial x})^2=\rho A\ddot{u}\end{aligned} \tag{6a}$$

$$AGk_1(\frac{\partial\varphi}{\partial x}+\frac{\partial^2v}{\partial x^2})+q+N\frac{\partial^2v}{\partial x^2}-kv=\rho A\ddot{v} \tag{6b}$$

$$AGk_2(\frac{\partial\psi}{\partial x}+\frac{\partial^2w}{\partial x^2})+p+N\frac{\partial^2w}{\partial x^2}-kv=\rho A\ddot{w} \tag{6c}$$

$$\begin{aligned}&I_zE_1\frac{\partial^2\varphi}{\partial x^2}+3\bar{I}_zE_3(\frac{\partial\varphi}{\partial x})^2\frac{\partial^2\varphi}{\partial x^2}+2I_zE_2\frac{\partial u}{\partial x}\frac{\partial^2\varphi}{\partial x^2}+2I_zE_2\frac{\partial\varphi}{\partial x}\times\\&\frac{\partial^2u}{\partial x^2}+6I_zE_3\frac{\partial u}{\partial x}\frac{\partial^2u}{\partial x^2}\frac{\partial\varphi}{\partial x}+3I_zE_3\frac{\partial^2\varphi}{\partial x^2}(\frac{\partial u}{\partial x})^2+6I_{yz}E_3\frac{\partial\psi}{\partial x}\times\\&\frac{\partial^2\psi}{\partial x^2}\frac{\partial\varphi}{\partial x}+3I_{yz}E_3(\frac{\partial\psi}{\partial x})^2\frac{\partial^2\varphi}{\partial x^2}-AGk_1(\varphi+\frac{\partial v}{\partial x})=\rho I_z\ddot{\varphi}\end{aligned} \tag{6d}$$

$$I_yE_1m^3n\frac{\partial^2\psi}{\partial x^2}+3\bar{I}_yE_3(\frac{\partial\psi}{\partial x})^2\frac{\partial^2\psi}{\partial x^2}+2I_yE_2\frac{\partial u}{\partial x}\frac{\partial^2\psi}{\partial x^2}+2I_yE_2\times$$
$$\frac{\partial\psi}{\partial x}\frac{\partial^2 u}{\partial x^2}+6I_yE_3\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2}\frac{\partial\psi}{\partial x}+3I_yE_3\frac{\partial^2\psi}{\partial x^2}(\frac{\partial u}{\partial x})^2+6I_{yz}E_3\times$$
$$\frac{\partial\psi}{\partial x}\frac{\partial^2\varphi}{\partial x^2}\frac{\partial\varphi}{\partial x}+3I_{yz}E_3(\frac{\partial\varphi}{\partial x})^2\frac{\partial^2\psi}{\partial x^2}-AGk_2(\psi+\frac{\partial w}{\partial x})=\rho I_y\ddot{\psi} \tag{6e}$$

可见，这是关于位移 u,v,w,φ,ψ 的一组耦合的非线性偏微分方程组.

边界条件由下面的虚功方程给出：

$$[AE_1\frac{\partial u}{\partial x}+AE_2(\frac{\partial u}{\partial x})^2+AE_3(\frac{\partial u}{\partial x})^3+I_zE_2(\frac{\partial\varphi}{\partial x})^2+$$
$$I_yE_2(\frac{\partial\psi}{\partial x})^2]\delta u|_0^l+[3I_zE_3(\frac{\partial\varphi}{\partial x})^2\frac{\partial u}{\partial x}+3I_yE_3(\frac{\partial\psi}{\partial x})^2\times$$
$$\frac{\partial u}{\partial x}\delta u]|_0^l-N\delta u(l)+[AGk_1(-\varphi-\frac{\partial v}{\partial x})+N\frac{\partial v}{\partial x}]\delta v|_0^l+$$
$$[AGk_2(-\psi-\frac{\partial w}{\partial x})+N\frac{\partial w}{\partial x}]\delta w|_0^l+[I_zE_1\frac{\partial\varphi}{\partial x}+\bar{I}_zE_3(\frac{\partial\varphi}{\partial x})^3+$$
$$2I_zE_2\frac{\partial\varphi}{\partial x}\frac{\partial u}{\partial x}+3I_zE_3(\frac{\partial u}{\partial x})^2\frac{\partial\varphi}{\partial x}+3I_{yz}E_3(\frac{\partial\psi}{\partial x})^2\times$$
$$\frac{\partial\varphi}{\partial x}]\delta\varphi|_0^l+[I_yE_1\frac{\partial\psi}{\partial x}+\bar{I}_yE_3(\frac{\partial\psi}{\partial x})^3+2I_yE_2\frac{\partial\psi}{\partial x}\frac{\partial u}{\partial x}+$$
$$3I_yE_3(\frac{\partial u}{\partial x})^2\frac{\partial\psi}{\partial x}+3I_{yz}\frac{4}{3}E_3(\frac{\partial\varphi}{\partial x})^2\frac{\partial\psi}{\partial x}]\delta\psi|_0^l=0 \tag{7}$$

如果桩的两端均为固定，则当 $x=1,l$ 时，$\delta u=\delta v=\delta w=\delta\varphi=\delta\psi$，因此，(7)式成立. 如果 $x=0$ 或者 $x=l$ 端自由或者给定端部力，则可以分别得到力端的边界条件. 例如，

(1) $x=l$ 端受轴拉力为 N 作用时的条件为

$$AE_1\frac{\partial u}{\partial x}+AE_2(\frac{\partial u}{\partial x})^2+AE_3(\frac{\partial u}{\partial x})^3+I_zE_2(\frac{\partial\varphi}{\partial x})^2+I_yE_2\times$$
$$(\frac{\partial\psi}{\partial x})^2+3I_zE_3(\frac{\partial\varphi}{\partial x})^2\frac{\partial u}{\partial x}+3I_yE_3(\frac{\partial\psi}{\partial x})^2\frac{\partial u}{\partial x}=N \tag{8a}$$

(2) $x=l$ 端不受剪力作用条件为

$$AGk_1(-\varphi-\frac{\partial v}{\partial x})+N\frac{\partial v}{\partial x}=0$$
$$AGk_2(-\psi-\frac{\partial w}{\partial x})+N\frac{\partial w}{\partial x}=0 \tag{8b}$$

(4) $x=l$ 端不受弯矩作用时的条件为

$$I_zE_1\frac{\partial\varphi}{\partial x}+\bar{I}_zE_3(\frac{\partial\varphi}{\partial x})^3+2I_zE_2\frac{\partial\varphi}{\partial x}\frac{\partial u}{\partial x}+$$
$$3I_zE_3(\frac{\partial u}{\partial x})^2\frac{\partial\varphi}{\partial x}+3I_{yz}E_3(\frac{\partial\psi}{\partial x})^2\frac{\partial\varphi}{\partial x}=0$$
$$I_yE_1\frac{\partial\psi}{\partial x}+\bar{I}_yE_3(\frac{\partial\psi}{\partial x})^3+2I_yE_2\frac{\partial\psi}{\partial x}\frac{\partial u}{\partial x}+$$
$$3I_yE_3(\frac{\partial u}{\partial x})^2\frac{\partial\psi}{\partial x}+3I_{yz}E_3(\frac{\partial\varphi}{\partial x})^2\frac{\partial\psi}{\partial x}=0 \tag{8c}$$

可以看到，由于材料的非线性，力端的端部条件也是非线性的. 因此，问题的求解是困难的，一般只能用数值方法求数值解.

初始条件：设桩基在 $t<0$ 处于自然状态，且当 $t\geq 0$ 时满足如下初始条件：

$$u|_{t=0}=u^0,\dot{u}|_{t=0}=\dot{u}^0,v|_{t=0}=v^0,\dot{v}|_{t=0}=\dot{v}^0$$
$$w|_{t=0}=w^0,\dot{w}|_{t=0}=\dot{w}^0,\varphi|_{t=0}=\varphi^0,\dot{\varphi}|_{t=0}=\dot{\varphi}^0$$
$$\psi|_{t=0}=\psi^0,\dot{\psi}|_{t=0}=\dot{\psi}^0 \tag{9}$$

其中，$u^0,\dot{u}^0,v^0,\dot{v}^0,w^0,\dot{w}^0,\varphi^0,\dot{\varphi}^0,\psi^0,\dot{\psi}^0$ 是仅与坐标 x 有关的已知函数，当初始时刻结构处于静止时，这些函数为零.

3　模型的应用与数值算例

3.1　非线性弹性桩基的轴向运动

(1)控制方程

如果我们只考虑结构轴向运动时，这时运动微分方程(6)、边界条件(7)和初始条件(8)将给出求解非线性弹性桩轴向运动的如下的无量纲形式的运动微分方程初边值问题，即

$$\frac{\partial^2 U}{\partial X^2}+E_{22}\frac{\partial U}{\partial X}\frac{\partial^2 U}{\partial X^2}+E_{33}(\frac{\partial U}{\partial X})^2\frac{\partial^2 U}{\partial X^2}=\frac{\partial^2 U}{\partial\tau^2} \tag{10a}$$

$$U|_{X=0}=0,[\frac{\partial U}{\partial X}+\frac{1}{2}E_{22}(\frac{\partial U}{\partial X})^2+\frac{1}{3}(\frac{\partial U}{\partial X})^3]|_{X=1}=N^* \tag{10b}$$

$$U|_{\tau=0}=0,\frac{\partial U}{\partial T}|_{\tau=1}=0 \tag{10c}$$

其中，无量纲量定义为：

$$X=\frac{x}{l},U=\frac{u}{l},E_{22}=\frac{2E_2}{E_1},E_{33}=\frac{3E_3}{E_1},N^*=\frac{N}{AE_1},$$
$$\tau=t\sqrt{E_1/(\rho l^2)} \tag{11}$$

(2)数值计算与结果分析

在空间上采用中心差分格式，在时间上采用向后差分格式，对(10)进行离散，对于每一个时间步长得到一组离散化的非线性代数方程，然后采用 Newton－Raphson 迭代方法求解离散化方程组，可以得到初边值问题(10)的数值解，数值计算结果示于图 2 中. 限于篇幅，这里省略了离散化的非线性代数方程组. 计算中的参数给定如下[12]：

$l=20\text{m},\rho=2.4\times10^3\text{kg/m}^3,E_1=2.1\times10^{10}\text{Pa}$、

$E_2=5.5\times10^{12}\text{Pa}$、$E_3=2.5\times10^{14}\text{Pa}$,

$A=1\text{m}^2(m=n=0.5\text{m}),N^*=0.005.$

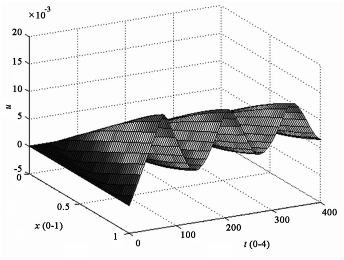

图 2　U 随 X 和 τ 的响应

Fig. 2　Response of U with X and τ

（3）参数的影响

由(11)看到，改变 E_1 的值将使相关参数发生变化. 当给定外载荷 N 时，取 E_1 为原来数值($E_1=2.1\times10^{10}$Pa)的 8 倍时，位移 U 随时间 τ 的变化曲线示于图 3 中. 可以看到，随着 E_1 的增大，位移将减小.

3.2　非线性桩基的平面耦合运动

（1）控制方程

令载荷 $q=0$ 时，则桩基在 xy - 平面内发生弯曲，这时有 $v=0,\varphi=0$. 运动方程(6)、边界条件(8)和初始条件(9)给出求解非线性桩基平面耦合运动的初边值问题.

无量纲形式的控制方程为

$$\begin{cases}\dfrac{\partial^2 U}{\partial X^2}+E_{22}\dfrac{\partial U}{\partial X}\dfrac{\partial^2 U}{\partial X^2}+E_{33}\left(\dfrac{\partial U}{\partial X}\right)^2\dfrac{\partial^2 U}{\partial X^2}+\dfrac{r}{3}E_{22}\dfrac{\partial\psi}{\partial X}\dfrac{\partial^2\psi}{\partial X^2}+\\ \quad\dfrac{2r}{3}E_{33}\dfrac{\partial\psi}{\partial X}\dfrac{\partial^2\psi}{\partial X^2}\dfrac{\partial U}{\partial X}+\dfrac{r}{3}E_{33}\dfrac{\partial^2 U}{\partial X^2}\left(\dfrac{\partial\psi}{\partial X}\right)^2=\dfrac{\partial^2 U}{\partial\tau^2}\\ G_1k_2\left(\dfrac{\partial\psi}{\partial X}+\dfrac{\partial^2 W}{\partial X^2}\right)+p^*+N^*\dfrac{\partial^2 W}{\partial X^2}-k^*W=\dfrac{\partial^2 W}{\partial X^2}\\ \dfrac{\partial^2\psi}{\partial X^2}+\dfrac{3r}{5}E_{33}\left(\dfrac{\partial\psi}{\partial X}\right)^2\dfrac{\partial^2\psi}{\partial X^2}+E_{22}\dfrac{\partial U}{\partial X}\dfrac{\partial^2\psi}{\partial X^2}+E_{22}\dfrac{\partial^2\psi}{\partial X^2}\dfrac{\partial U}{\partial X}+\\ 2E_{33}\dfrac{\partial U}{\partial X}\dfrac{\partial^2 U}{\partial X^2}\dfrac{\partial\psi}{\partial X}+E_{33}\dfrac{\partial^2\psi}{\partial X^2}\left(\dfrac{(\partial U}{\partial X}\right)^2-\dfrac{3}{r}G_1k_2\left(\psi+\dfrac{\partial W}{\partial X}\right)=\dfrac{\partial^2\psi}{\partial\tau^2}\end{cases}\tag{12a}$$

端部条件为

$$U=0,\quad W=0,\quad \frac{\partial\psi}{\partial X}=0,\quad X=0$$

$$\frac{\partial U}{\partial X}+\frac{1}{2}E_{22}\left(\frac{\partial U}{\partial X}\right)^2+\frac{1}{3}E_{33}\left(\frac{\partial U}{\partial X}\right)^3+\frac{r}{6}E_{22}\left(\frac{\partial\psi}{\partial X}\right)^2+$$

$$\frac{r}{3}E_{33}\left(\frac{\partial\psi}{\partial X}\right)^2\frac{\partial U}{\partial X}=N^*,X=1$$

$$W=0,\quad\frac{\partial\psi}{\partial X}=0,\quad X=1\tag{12b}$$

这里，已设 $x=0$ 端简支，$x=l$ 受拉力的作用，并在 z 方向固定. 初始条件为

$$U=\frac{\partial U}{\partial\tau}=0,W=\frac{\partial W}{\partial\tau}=0,\psi=\frac{\partial\psi}{\partial\tau}=0\tag{12c}$$

其中，无量纲参数定义如下：

$$X=\frac{x}{l},U=\frac{u}{l},W=\frac{w}{l},\psi=\Psi,E_{22}=\frac{2E_2}{E_1},E_{33}=\frac{3E_3}{E_1},$$

$$k^*=\frac{kl}{E_1A},N^*=\frac{N}{AE_1},p^*=\frac{pl}{E_1A},G_1=\frac{G}{E_1},r=\frac{m^2}{l^2},$$

$$\tau=t\sqrt{E_1/(\rho l^2)}\tag{13}$$

（2）数值计算与结果分析

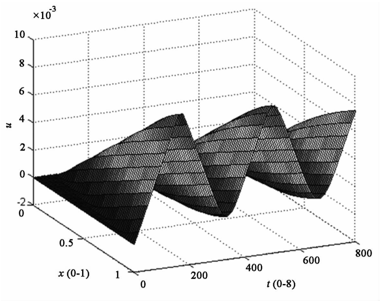

图 3　U 随 X 和 τ 的响应

Fig. 3　Response of U with X and τ

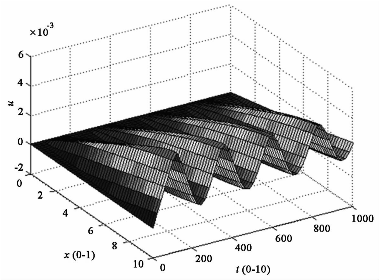

图 4　U 随 X 和 τ 的响应

Fig. 4　Response of U with X and τ

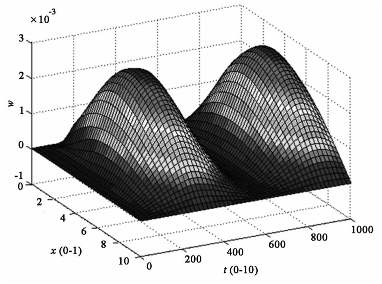

图 5　W 随 X 和 τ 的响应

Fig. 5　Response of W with X and τ

同样，在空间上采用中心差分格式，在时间上采用向后差分格式对(12)进行离散，对每一个时间步长得到一组关于 U,W,ψ 的离散化的非线性代数方程，采用 Newton - Raphson 迭代方法求解离散化方程组，可以得到初边值问题(12)的数值解，图 3 - 图 5 分别示出了 U,W,ψ 随时间 τ 的变化. 限于篇幅，这里也省略了离散化的非线性代数方程组. 计算中的参数给定如下[12]：

$l=20\mathrm{m}$,$\rho=2.4\times10^3\mathrm{kg/m^3}$,$E_1=2.1\times10^{10}\mathrm{Pa}$、
$E_2=5.5\times10^{12}\mathrm{Pa}$、$E_3=2.5\times10^{14}\mathrm{Pa}$,
$A=1\mathrm{m}^2$,$N^*=0.005$,$p^*=0.001$,$k=0$.

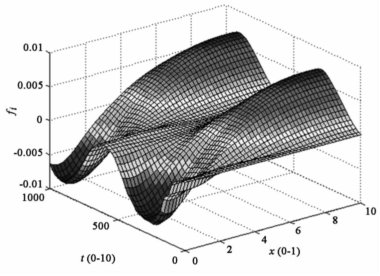

图6 ψ 随 X 和 τ 的响应

Fig. 6 Response of ψ with X and τ

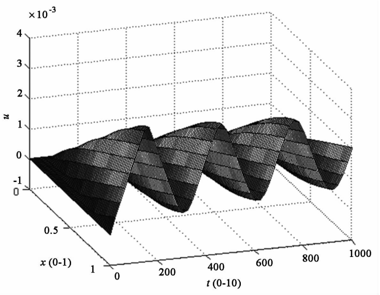

图7 U 随 X 和 τ 的响应

Fig. 7 Response of U with X and τ

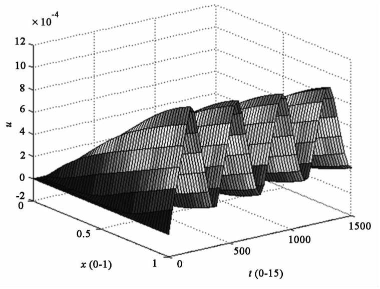

图8 U 随 X 和 τ 的响应

Fig. 8 Response of U with X and τ

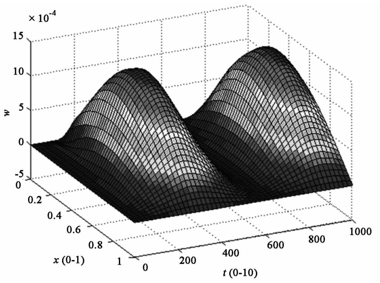

图9 W 随 X 和 τ 的响应

Fig. 9 Response of W with X and τ

(3)参数的影响

根据(13),当给定外载荷 P 和 N 时,取 E_1 为原来数值($E_1=2.1\times10^{10}\mathrm{Pa}$)的2,8倍时,$U$,$W$,$\psi$ 随时间 τ 的变化曲线示于图7－图12中.可以看到,随着 E_1 的增大,位移和转角都将减小.

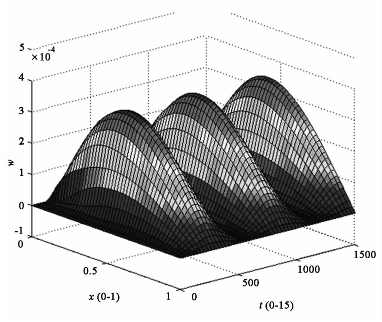

图10 W 随 X 和 τ 的响应

Fig. 10 Response of W with X and τ

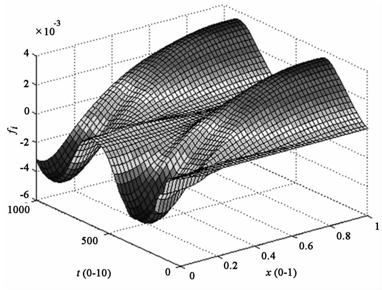

图11 ψ 随 X 和 τ 的响应

Fig. 11 Response of ψ with X and τ

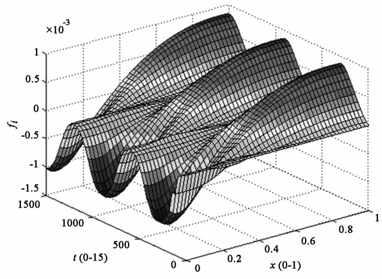

图12 ψ 随 X 和 τ 的响应

Fig. 12 Response of ψ with X and τ

4 结论

本文根据Timoshenko梁的假设和Hamilton原理,建立了小变形条件下,3次非线性弹性材料桩基的动力学行为分析的数学模型.作为该非线性数学模型的应用,利用有限差分法对非线性弹性桩基的轴向运动和平面耦合运动的控制方程和边界条件进行了离散,然后采用Newton-Raphson迭代方法求解离散化的非线性代数方程组,并由此得到了问题的数值解,因此,得到了位移随时间和空间的变化.考察了非线性弹性性质(即广义弹性模量)对结构动力学响应的影响.可以看到,非线性弹性桩基的轴向运动和平面耦合运动均是周期运动,非线性弹性性质增加使得运动的幅值减小,速率变缓.

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DYNAMIC RESPONSE ANALYSIS OF NONLINEAR PILES

Hua Yuchao[1] Zhu Yuanyuan[2] Cheng Changjun[1]

(1. *Shanghai Institute of Applied Mathematic and Mechanics*, *Department of Mechanics*, *Shanghai University*, *Shanghai* 200072, *China*)

(2. *Department of Computer Science and Technology*, *Shanghai Normal University*, *Shanghai* 200234, *China*)

Abstract The material of piles was assumed as one of the nonlinear elastic materials, which obeys a thrice nonlinear constitutive relation, and the pile was located on an elastic foundation. Following Timoshenko theory of beams and the Hamilton variational principle, a mathematical model of dynamic behavior analysis of nonlinear elastic piles was established, in which three displacements and two angles were contained. As application of the model, the axial movement and the plane coupling movement of nonlinear elastic piles were computed by using the finite difference method and Newton - Raphson iteration method. The corresponding numerical solutions were presented. The effects of the nonlinear elastic properties on the structural dynamic response were considered.

Key words nonlinear elastic material, Hamilton variational principle, coupling movement of nonlinear piles, dynamic response, effect of nonlinear elastic parameter

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