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集中谐载力作用下三边固定一边自由板的受迫振动

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摘要　应用混合变量最小作用量原理求解了三边固定一边自由矩形板在任意集中谐载作用下的受迫振动问题,建立了求解这类问题受迫振动稳态解的新方法.对于结构工程抗震问题中的振动分析给出了一个新的解决途径.所获得的计算结果,跟现有的文献进行比较证明了其正确性.因此该方法不仅具有重要的理论价值,而且可以为工程实际直接采用.

关键词　混合变量,最小作用量,任意集中谐载,受迫振动,稳态解

引言

弯曲矩形薄板在工程实际中有着广泛的应用,板的经典理论用于计算薄板的弯曲、稳定与振动时会产生较大的误差.在以往的解法中,都难以找到一种简便的位移函数来求解它.本文中应用文献[1]中的混合变量最小作用量原理只要求位移是弱容许的,即只要求位移预先满足应变-位移关系,而不要求位移预先满足位移边界条件.这就使得求解这类问题时不需要考虑复杂位移的假设,根据弯曲矩形薄板的实际边界条件即可建立起混合变量的总势能从而求出受迫振动问题的稳态解.从而克服了经典解法的繁冗计算过程和对某些问题难以求解等局限性.

1　基本方程

图1(a)为三边固定一边自由矩形板,解除固定边 $x=0, y=0, x=a$ 的弯曲约束代以弯矩幅值 $\overline{M}_{x0}\overline{M}_{xa}\overline{M}_{y0}$ 如图1(b)所示,同时假设自由边 $y=b$ 的挠度幅值 $\overline{w}_{yb}$

$$\overline{M}_{x0}=\sum_{n=1,2}^{\infty}A_n\sin\beta_n y \tag{1}$$

$$\overline{M}_{xa}=\sum_{n=1,2}^{\infty}B_n\sin\beta_n y \tag{2}$$

$$\overline{M}_{y0}=\sum_{m=1,2}^{\infty}C_m\sin\frac{m\pi x}{a} \tag{3}$$

$$\overline{w}_{yb}=\sum_{m=1,2}^{\infty}d_m\sin\frac{m\pi x}{a} \tag{4}$$

其中 $\beta_n=\frac{n\pi}{b}$, A_n, B_n, c_m, d_m 为待定系数.

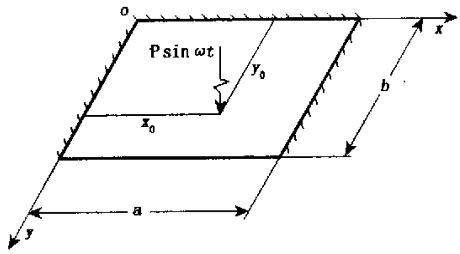

图1(a)　简谐集中荷载三边固定一边自由矩形板

Fig.1(a)　The rectangular plate with three clamped and the other free under concentrated load

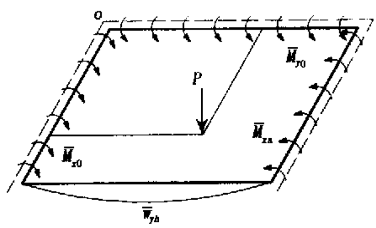

图1(b)　矩形板幅值实际系统

Fig.1(b)　The actual system of amplitude

设任意集中谐载

$$F(x,y,t)=P\delta(x-x_0,y-y_0)\sin\omega t$$

边界条件为

$$\begin{cases}\left(\frac{\partial w}{\partial y}\right)_{y=0}=0,\left(\frac{\partial w}{\partial x}\right)_{x=0,a=0}=0\\ \left[\frac{\partial^3 w}{\partial y^3}+(2-v)\frac{\partial^3 w}{\partial x^2\partial y}\right]_{y=0}=0\end{cases} \tag{5}$$

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$$\begin{cases} w_{x=0} = w_{x=a} = w_{y=0} = 0 \\ \left(\dfrac{\partial^2 w}{\partial y^2} + v\dfrac{\partial^3 w}{\partial x^2}\right)_{y=b} = 0 \end{cases} \tag{6}$$

2 求解任意集中谐载作用下三边固定一边自由矩形板的受迫振动

混合变量幅值总势能为

$$\prod_{mp} = \int_0^a\int_0^b \frac{1}{2}D\left\{\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)^2 - 2(1-v)\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x\partial y}\right)^2\right]\right\}dxdy - \int_0^a\int_0^b\left(Pw + \frac{1}{2}\rho\omega^2 w^2\right)dxdy + \int_0^a \overline{w}_{yb}(V_y)_{y=b}dx - \int_0^b \overline{M}_{x0}\left(\frac{dw}{dx}\right)_{x=0}dy + \int_0^b \overline{M}_{xa}\left(\frac{dw}{dx}\right)_{x=a}dy - \int_0^a \overline{M}_{y0}\left(\frac{dw}{dy}\right)_{y=0}dy \tag{7}$$

假设弯曲矩形板的挠曲面方程为

$$w(x,y) = \sum_{m=1,2}^{\infty}\sum_{n=1,2}^{\infty} A_{mn}\sin\alpha_m x\sin\beta_n y \quad (0 \leqslant x \leqslant a \quad 0 \leqslant y < b) \tag{8}$$

其中 $\alpha_m = \frac{m\pi}{a}$. 将幅值挠曲面式(7)代入式(8)取变分极值经计算,于是有

$$\delta\prod_{mp} = \frac{Dab}{4}\sum_{m=1,2}^{\infty}\sum_{n=1,2}^{\infty} A_{mn}\delta A_{mn}K_{dmn}^2 - q\sum_{m=1,2}^{\infty}\sum_{n=1,2}^{\infty}\delta A_{mn}\frac{ab}{\pi^2 mn}[1-(-1)^m]\times[1-(-1)^n] - \sum_{m=1,2}^{\infty}\sum_{n=1,2}^{\infty}\frac{b}{2}A_n\alpha_m\delta A_{mn} - \sum_{m=1,2}^{\infty}\sum_{n=1,2}^{\infty}(-1)^m\frac{b}{2}A_n\alpha_m\delta A_{mn} + \sum_{m=1,2}^{\infty}\sum_{n=1,2}^{\infty}\frac{a}{2}C_m\beta_n\delta A_{mn} + (-1)^n\frac{Da}{2}\left[\sum_{m=1,2}^{\infty}\sum_{n=1,2}^{\infty}\delta A_{mn}d_m\beta_n^3 + (2-v)\sum_{m=1,2}^{\infty}\sum_{n=1,2}^{\infty}\delta A_{mn}d_m\alpha_m^2\beta_n\right] = 0 \tag{9}$$

其中 $\lambda^2 \equiv \frac{1}{D}\rho\omega^2, K_{dmn}^2 \equiv (\alpha_m^2+\beta_n^2)^2 - \lambda^2$. 由式(9)可求得

$$A_{mn} = \sum_{m=1,2}^{\infty}\sum_{n=1,2}^{\infty}\frac{4q}{D\pi^2}\frac{1}{mn}\frac{1}{K_{dmn}^2}[1-(-1)^m]\times[1-(-1)^n] + \sum_{m=1,2}^{\infty}\sum_{n=1,2}^{\infty}\frac{2}{Da}\frac{\alpha_m}{K_{dmn}^2}(A_n) + \sum_{m=1,2}^{\infty}\sum_{n=1,2}^{\infty}\frac{(-1)^n 2}{Da}\frac{\alpha_m}{K_{dmn}^2}(B_n) + \sum_{m=1,2}^{\infty}\sum_{n=1,2}^{\infty}\frac{2}{Db}\frac{\beta_n}{K_{dmn}^2}(C_m) - \sum_{m=1,2}^{\infty}\sum_{n=1,2}^{\infty}\frac{2}{b}\frac{2(-1)^n}{K_{dmn}^2}\beta_n[\beta_n^2 + (2-v)\alpha_m^2](d_m) \tag{10}$$

将式(10)代入式(8)三角级数表示的分挠曲面幅值方程分别为

$$w_1 = \frac{4P}{Dab}\sum_{m=1,2}^{\infty}\sum_{n=1,2}^{\infty}\frac{\sin\alpha_m x_0\sin\beta_n y_0}{K_{dmn}^2}\times\sin\alpha_m x\sin\beta_n y \tag{11}$$

$$w_2 = \frac{2}{Da}\sum_{m=1,2}^{\infty}\sum_{n=1,2}^{\infty}\frac{\alpha_m}{K_{dmn}^2}\sin\alpha_m x\sin\beta_n y(A_n) \tag{12}$$

$$w_3 = -\frac{2}{Da}\sum_{m=1,2}^{\infty}\sum_{n=1,2}^{\infty}\frac{(-1)^m\alpha_m}{K_{dmn}^2}\times\sin\alpha_m x\sin\beta_n y(B_n) \tag{13}$$

$$w_4 = \frac{2}{Db}\sum_{m=1,2}^{\infty}\sum_{n=1,2}^{\infty}\frac{\beta_n}{K_{dmn}^2}\sin\alpha_m x\sin\beta_n y(C_m) \tag{14}$$

$$w_5 = -\frac{2}{b}\sum_{m=1,2}^{\infty}\sum_{n=1,2}^{\infty}\frac{(-1)^n}{K_{dmn}^2}[\beta_n^3 + \alpha_m^2\beta_n(2-v)]\sin\alpha_m x\sin\beta_n y(d_m) \tag{15}$$

为了加快收敛速度和消除三角级数表示的挠度和弯矩幅值在边界上所出现的第一类间断点,还必须将分挠曲面幅值方程表示为三角级数和双曲函数混合表示的形式.

对于 $\alpha_m^2 < \lambda$ 和 $\beta_n^2 < \lambda$ 的情况,则有

$$w_1 = \frac{2P}{Db}\sum_{n=1,2}^{\infty}\frac{1}{\alpha_n'^2 - \beta_n'^2}\left[-\frac{\text{sh}\alpha_n'(a-x_0)\text{sh}\alpha_n' x}{\alpha_n'\text{sh}\alpha_n' a} + \frac{\text{sh}\beta_n'(a-x_0)\text{sh}\beta_n' x}{\beta_n'\text{sh}\beta_n' a}\right]\sin\beta_n y_0\sin\beta_n y \quad 0 \leqslant x \leqslant x_0 \tag{16}$$

$$w_1 = \frac{2P}{Db}\sum_{n=1,2}^{\infty}\frac{1}{\alpha_n'^2 - \beta_n'^2}\left[-\frac{\text{sh}\alpha_n' x_0\text{sh}\alpha_n'(a-x)}{\alpha_n'\text{sh}\alpha_n' a} + \frac{\text{sh}\beta_n' x_0\text{sh}\beta_n'(a-x)}{\beta_n'\text{sh}\beta_n' a}\right]\sin\beta_n y_0\sin\beta_n t \quad x_0 \leqslant x \leqslant a \tag{17}$$

$$w_1 = \frac{2P}{Db}\sum_{n=1,2}^{\infty}\frac{1}{\alpha_m'^2 - \beta_m'^2}\left[-\frac{\text{sh}\alpha_m'(b-y_0)\text{sh}\alpha_m' y}{\alpha_m'\text{sh}\alpha_m' b} + \frac{\text{sh}\beta_m'(b-y_0)\text{sh}\beta_m' y}{\beta_m'\text{sh}\beta_m' b}\right]\sin\alpha_m x_0\sin\alpha_m x$$

$$0 \leqslant y \leqslant y_0 \quad (18)$$

$$w_1 = \frac{2P}{Db}\sum_{n=1,2}^{\infty}\frac{1}{\alpha_m'^2 - \beta_m'^2}\Big[-\frac{\mathrm{sh}\alpha_m'\mathrm{sh}\alpha_M'(b-y)}{\alpha_m'\mathrm{sh}\alpha_m' b} + \frac{\mathrm{sh}\beta_m' y_0\mathrm{sh}\beta_m'(b-y)}{\beta_m'\mathrm{sh}\beta_m' b}\Big]\sin\alpha_m x_0\sin\alpha_m x$$

$$y_0 \leqslant y \leqslant b \quad (19)$$

其中 $\alpha_n'^2 = \sqrt{\beta_n^2 + \lambda}$, $\beta_n'^2 = \sqrt{\beta_n^2 - \lambda}$, $\alpha_m'^2 = \sqrt{\alpha_m^2 + \lambda}$, $\beta_m'^2 = \sqrt{\alpha_n^2 - \lambda}$.

$$w_2 = \frac{1}{D}\sum_{n=1,2}^{\infty}\frac{1}{\alpha_n'^2 - \beta_n'^2}\Big[-\frac{\mathrm{sh}\alpha_n'(a-x)}{\mathrm{sh}\alpha_n' a} + \frac{\mathrm{sh}\beta_n'(a-x)}{\mathrm{sh}\beta_n' a}\Big]\sin\beta_n y(A_n) \quad (20)$$

$$w_3 = \frac{1}{D}\sum_{m=1,2}^{\infty}\frac{1}{\alpha_m'^2 - \beta_m'^2}\Big[\frac{\mathrm{sh}\alpha_n' x}{\mathrm{sh}\alpha_m' a} - \frac{\mathrm{sh}\beta_m' x}{\mathrm{sh}\beta_m' a}\Big]\sin\beta_n y(B_n) \quad (21)$$

$$w_4 = \frac{1}{D}\sum_{m=1,2}^{\infty}\frac{1}{\alpha_m'^2 - \beta_m'^2}\Big[-\frac{\mathrm{sh}\alpha_m'(b-y)}{\mathrm{sh}\alpha_m' b} + \frac{\mathrm{sh}\beta_m'(b-y)}{\mathrm{sh}\beta_m' b}\Big]\sin\alpha_m x(C_n) \quad (22)$$

$$w_5 = \sum_{m=1,2}^{\infty}\frac{1}{\alpha_m'^2 - \beta_m'^2}\Big\{[\alpha_m'^2 - \alpha_m^2(2-v)]\frac{\mathrm{sh}\alpha_n' y}{\mathrm{sh}\alpha_n' b} - [\beta_m'^2 - \alpha_m^2(2-v)]\frac{\mathrm{sh}\beta_m' y}{\mathrm{sh}\beta_m' b}\Big\}\sin\alpha_m x(d_m) \quad (23)$$

当 $\alpha_m^2 < \lambda$ 和 $\beta_m^2 < \lambda$ 时,易于得到相应的分挠曲面幅值方程,由于篇幅有限不再给出了.

考察边界条件.边界条件式(5)已自动满足,下面考察边界条件式(6)

当 $\alpha_m^2 < \lambda$ 和 $\beta_m^2 < \lambda$ 时,执行边界条件 $\left(\frac{\partial w}{\partial y}\right)_{y=0} = 0$,则得

$$\frac{2P}{Da}\Big[-\frac{\mathrm{sh}\alpha_m'(b-y_0)}{\mathrm{sh}\alpha_m' b} + \frac{\mathrm{sh}\beta_m'(b-y_0)}{\mathrm{sh}\beta_m' b}\Big]\times \sin\alpha_m x_0 + \frac{4\lambda}{Da}\sum_{n=1,2}^{\infty}\frac{\alpha_m\beta_n}{K_{dmn}^2}(A_n) - \frac{4\lambda}{Da}\sum_{n=1,2}^{\infty}\frac{(-1)^m\alpha_m\beta_n}{K_{dmn}^2}(B_n) + \frac{1}{D}(\alpha_m'\mathrm{cth}\alpha_m' b - \beta_m\mathrm{cth}\beta_m' b)(C_m) + \Big\{[\alpha_m'^2 - \alpha_m^2(2-v)]\frac{\alpha_m}{\mathrm{sh}\alpha_m' b} - [\beta_m'^2 - \alpha_m^2(2-v)]\frac{\beta_m'}{\mathrm{sh}\beta_m' b}\Big\}(d_m) = 0 \quad (24)$$

边界条件 $\left(\frac{\partial w}{\partial x}\right)_{y=0} = 0$,得

$$\frac{2P}{Db}\Big(-\frac{\mathrm{sh}\alpha_n'(a-x_0)}{\mathrm{sh}\alpha_n' a} + \frac{\mathrm{sh}\beta_n'(a-x_0)}{\mathrm{sh}\beta_n' a}\Big]\sin\beta_n y_0 + \frac{1}{D}(\alpha_n'\mathrm{cth}\alpha_n' a - \beta_n'\mathrm{cth}\beta_n' a)(A_m) - \frac{1}{D}\Big(\frac{\alpha_n'}{\mathrm{sh}\alpha_n' a} - \frac{\beta_n'}{\mathrm{sh}\beta_n' a}\Big)(B_n) + \frac{4\lambda}{Db}\times \sum_{m=1,2}^{\infty}\frac{\alpha_m\beta_n}{K_{dmn}^2}(C_n) - \frac{4\lambda}{b}\sum_{m=1,2}^{\infty}\frac{(-1)^n\alpha_m\beta_n}{K_{dmn}^2}\times [\beta_m^2 + \alpha_m^2(2-v)](d_m) = 0 \quad (25)$$

边界条件 $\left[\frac{\partial^3 w}{\partial y^3} + (2-v)\frac{\partial^3 w}{\partial x^2\partial y}\right]_{y=b} = 0$,得

$$\frac{2P}{Da}\Big\{[\alpha_m'^2 - \alpha_m^2(2-v)]\frac{\mathrm{sh}\alpha_m}{\mathrm{sh}\alpha_m' b} - [\beta_m'^2 - \alpha_m^2(2-v)]\frac{\mathrm{sh}\beta_m}{\mathrm{sh}\beta_m' b}\Big\}\sin\alpha_m x_0 - \frac{4\lambda}{Da}\sum_{n=1,2}^{\infty}\frac{(-1)^n\alpha_m\beta_n}{K_{dmn}^2}[\alpha_m^2 + \beta_m^2(2-v)](A_n) + \frac{4\lambda}{Da}\sum_{n=1,2}^{\infty}\frac{(-1)^{m+n}\alpha_m\beta_n}{K_{dmn}^2}[\alpha_m^2 + \beta_m^2(2-v)](B_n) + \frac{1}{D}\Big\{[\alpha_m'^2 - \alpha_m^2(2-v)]\frac{\alpha_m}{\mathrm{sh}\alpha_m' b}[\beta_m'^2 - \alpha_m^2(2-v)]\frac{\beta_m}{\mathrm{sh}\beta_m' b}\Big\}(C_m) + \{[\alpha_m'^2 - \alpha_m^2(2-v)]^2\alpha_m'\mathrm{cth}\alpha_m' b - [\beta_m'^2 - \alpha_m'^2(2-v)]^2\beta_m'\mathrm{cth}\beta_m' b\} = 0 \quad (26)$$

当 $\alpha_m^2 < \lambda$ 和 $\beta_n' < \lambda$ 时,边界条件式(24)~式(26)分别为

$$\frac{2P}{Da}\Big[-\frac{\mathrm{sh}\alpha_m'(b-y_0)}{\mathrm{sh}\alpha_m' b} + \frac{\mathrm{sh}\beta_m(b-y_0)}{\mathrm{sh}\beta_m' b}\Big]\times \sin\alpha_m x_0 + \frac{4\lambda}{Da}\sum_{n=1,2}^{\infty}\frac{\alpha_m\beta_n}{K_{dmn}^2}(A_n) - \frac{4\lambda}{Da}\sum_{n=1,2}^{\infty}\frac{(-1)^m\alpha_m\beta_n}{K_{dmn}^2}(B_n) + \frac{1}{D}(\alpha_m'\mathrm{cth}\alpha_m' b - \beta_m\mathrm{ctg}\beta_m'' b)(C_m) + \Big\{[\alpha_m'^2 - \alpha_m^2(2-v)]\frac{\alpha_m}{\mathrm{sh}\alpha_m' b} + [\beta_m''^2 - \alpha_m^2(2-v)]\frac{\beta_m''}{\mathrm{sh}\beta_m'' b}\Big\}(d_m) = 0 \quad (27)$$

$$\frac{2P}{Db}\Big(-\frac{\mathrm{sh}\alpha_n'(a-x_0)}{\mathrm{sh}\alpha_n' a} + \frac{\mathrm{sh}\beta_n''(a-x_0)}{\mathrm{sh}\beta_n'' a}\Big]\sin\beta_n y_0 + \frac{1}{D}(\alpha_n'\mathrm{cth}\alpha_n' a - \beta_n''\mathrm{cth}\beta_n'' a)(A_n) -$$

$$\frac{1}{D}\left(\frac{\alpha_n'}{\mathrm{sh}\alpha_n' a}-\frac{\beta_n''}{\mathrm{sh}\beta_n'' a}\right)(B_n)+\frac{4\lambda}{Db}\times\sum_{m=1,2}^{\infty}\frac{\alpha_m\beta_n}{K_{dmn}^2}(C_n)-\frac{4\lambda}{b}\sum_{m=1,2}^{\infty}\frac{(-1)^n\alpha_m\beta_n}{K_{dmn}^2}\times[\beta_m^2+\alpha_m^2(2-v)](d_m)=0 \quad (28)$$

$$\frac{2P}{Da}\left\{[\alpha_m'^2-\alpha_m^2(2-v)]\frac{\mathrm{sh}\alpha_m' y_0}{\mathrm{sh}\alpha_m' b}-\left[\beta_m''^2-\alpha_m^2(2-v)\frac{\mathrm{sh}\beta_m'' y_0}{\mathrm{sh}\beta_m'' b}\right]\right\}\sin\alpha_m x_0-\frac{4\lambda}{Da}\sum_{n=1,2}^{\infty}\frac{(-1)^n\alpha_m\beta_n}{K_{dmn}^2}[\alpha_m^2+\beta_n^2(2-v)](A_n)+\frac{4\lambda}{Da}\sum_{n=1,2}^{\infty}\frac{(-1)^{m+n}\alpha_m\beta_n}{K_{dmn}^2}[\alpha_m^2+\beta_n^2(2-v)](B_n)+\frac{1}{D}\left\{[\alpha_m'^2-\alpha_m^2(2-v)]\frac{\alpha_m'}{\mathrm{sh}\alpha_m' b}-[\beta_m''^2-\alpha_m^2(2-v)]\frac{\beta_m''}{\mathrm{sh}\beta_m'' b}\right\}(C_m)+\{[\alpha_m'^2-\alpha_m^2(2-v)]^2\mathrm{cth}\alpha_m' b-\beta_m''[\beta_m''^2-\alpha_m'^2(2-v)]^2\mathrm{cth}\beta_m'' b\}(d_m)=0 \quad (29)$$

3 数值计算分析

于是我们得到四组无穷联立式(24)式(26)或式(27)和式(29).取出限项.解出 A_n, B_n, C_m 和 d_m 进而可按式(2)和式(4)求得固定边的弯矩幅值和自由边的挠度幅值.具体地,取 $v=1/6, a/b=1$. A_n, B_n, C_m 和 d_m 各取15项,并假设集中谐载作用于板的中点,编程计算,对于不同的 ω/ω_{11} 值,得到固定边的弯矩幅值和弯矩边的挠度幅值见表1,以及如图2和图3所示.

表1 固定边弯矩幅值(P)和最大挠度幅值(Pa^2/D) $a/b=1$

Table 1 The amplitudes(P)of the moment and the maximum deflection (Pa^2/D) $a/b=1$at the clamped ends

ω/ω_{11}	$M(w)$	$x/a(y/b)$						
		0.05	0.15	0.35	0.5	0.7	0.9	0.95
	M_{x0}	0.000647	-0.02006	-0.1066	-0.1497	-0.13490	-0.08149	-0.06926
0.0	M_{y0}	0.001470	-0.01934	-0.1045	-0.1355	-0.08397	-0.00485	0.00147
	w_{yb}	0.000044	0.000508	0.002086	0.002634	0.001712	0.000216	0.000044
	M_{x0}	0.000359	0.022110	-0.1142	-0.16140	-0.14970	-0.09592	-0.08865
0.3	M_{y0}	0.001633	-0.02097	-0.1122	-0.1451	-0.09023	-0.00534	0.001633
	w_{yb}	0.000062	0.000634	0.002515	0.00316	0.002074	0.000278	0.000062
	M_{x0}	-0.000416	-0.02682	-0.13110	0.1879	-0.18500	-0.13200	-0.13840
0.5	M_{y0}	0.002043	-0.02459	-0.12920	-0.1664	-0.1042	-0.00641	0.002043
	w_{yb}	0.000106	0.000957	0.003601	0.004489	0.002989	0.000435	0.000106
	M_{x0}	-0.00697	-.05466	-0.22210	-0.34180	-0.4176	-0.3949	-0.5209
0.8	M_{y0}	0.00512	-0.04353	-.2211	-0.2824	-0.17911	-0.01179	0.00512
	w_{yb}	0.000457	0.003413	0.01171	0.01438	0.009844	0.001648	0.000457

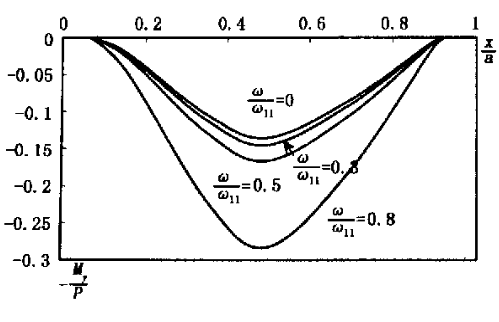

图2 固定边 $y=0$ 弯矩幅值分布, $a/b=1$

Fig.1 The amplitudes $a/b=1$ of the moment of clamped edge $y=0$

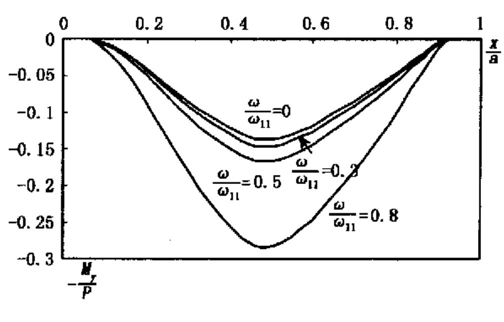

图3 自由边 $y=b$ 弯矩幅值分布, $a/b=1$

Fig.2 The amplitudes $a/b=1$ of the moment of free edge $y=b$

讨论:

(1) 关于系数收敛性问题. 由于载荷和结构对于平衡于 y 方向轴呈对称性,故有 $A_n = B_n, C_m = d_m = 0(m = 2,4,\cdots)$,即 C_m 和 d_m 实际上只取八项. 而且计算发现,A_n 从 10^0 数量级收敛到 10^{-2} 数量级, C_m 从 10^0 数量级收敛到 10^{-4} 数量级, 从 10^{-2} 数量级收敛到 10^{-5} 数量级,这比均布谐载的收敛要好一些.

(2) 关于 M_{x0} 分布. 计算发现,当 $\omega/\omega_{11} = 0.8$ 时,靠近自由边端部将出现较大的 M_{x0} 值. 这表明,自由边对其邻边弯矩幅值的影响随着载荷频率接近固有频率而显著增加.

(3) 关于 M_{x0} 分布. 当 ω/ω_{11} 取值不同值时,M_{y0} 沿 $y = 0$ 边呈光滑对称分布,且在两端部有微小的反向弯矩出现.

(4) 载荷频率的影响. 当 ω/ω_{11} 从 0.0 增加到 0.8 时,自由边中点挠度幅值增加约 4.46 倍, 沿 $y = 0$ 固定边中点最大弯矩幅值增加约一倍.

4　结论

1. 本文应用混合变量法求解了三边固定一边自由矩形板在任意集中谐载作用下的受迫振动问题,得到其受迫振动的稳态解.

2. 混合变量法是求解弯曲矩形薄板受迫振动问题的一种简单、通用、有效的新方法.

3. 应用混合变量法所获得的计算结果是正确的,可以为工程实际直接采用.

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THE FORCED VIBRATION OF THE PLATE WITH THREE CLAMPED AND THE OTHER FREE UNDER CONCENTRATED LOAD

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Abstract　The principle of least action with variables was used to solve the forced vibration of the rectangular plate with three clamped and the other free under concentrated load, and the corresponding stable solution was obtained. Moreover, the principle was extended to solve the forced vibration of straight stem and bending of thin rectangular plates, and a new method for solving this kind of problems was established. The results were compared with the literates, which proved our method to be true. So the method gives us a new way to solve some problems about earthquake resistance and vibration analysis of the architecture engineering.

Key words　mixed variables, least action, concentrated load, forced vibration, stable solution

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